Precalculus
Worksheet 9.1

Counting Without Formulas … Tree Diagrams and Lists

1. Dinner at a restaurant is advertised as follows:

Make a tree diagram that shows every possible dinner possible.

Dinner … $8.95 … Choose 1 item from each Column		
Appetizer	Entrée	Dessert
Soup Salad	Baked Chicken Broiled Beef Baby Beef Liver Roast Beef Au Jus	Ice Cream Cheese Cake

2. You are going to play 3 games in the next three days. Make a list of all possible outcomes if each game will end in either a Win or a Loss.

3. Go back and apply the multiplication counting principle to questions 1 and 2.

4. When you go to Chipotle to make a burrito you have the following choices:

Meat	Rice	Beans	Salsas	Cheese	Sour Cream
Chicken Steak Pork Beef Veggie (no meat)	Yes No	Refried Pinto None	Mild Medium Medium-Hot Hot None	Yes No	Yes No

Use the multiplication counting principle to count the number of possible burritos that can be made assuming you select one choice from each category above.

5. You need to create a 4-digit PIN number for your ATM card. (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

   a) How many different passwords can be made if repetition of digits is allowed?

   b) How many different passwords can be made if repetition of digits is not allowed?

6. Twenty students are semi-finalists for four scholarships -- one for $1500, one for $1000, one for $800, and one for $500. How many different ways can the scholarships be awarded if no one person gets more than one scholarship?

   a) Use the multiplication counting principle to answer this question.

   b) You may also use permutations to solve this problem. Why?

   c) Use permutations to answer this question.

   d) Based on your answer to part *a* and *c*, what is the quickest way to calculate ${}_7P_3$ without a calculator?

7. A menu at a Chinese restaurant contains 48 main dishes. A group of friends decides to order 6 different dishes.

   a) This problem can be solved using combinations. Why?

   b) In how many different ways can the group order the 6 different dishes.

8. Applebees offers a "2 for $20" deal that involves selecting 1 Appetizer and 2 entrées for $20.

   a) Assuming the same entrée could be ordered by both people, how many different "2 for $20" meals can be made?

   b) How many different "2 for $20" meals can be made if we assume each person will order a different entrée?

Appetizers	Entrée
Spinach & Artichoke Dip Mozzarella Sticks Crunchy onion rings Potato twisters	Brushetta Chicken Chicken Fettuccine Carbonara 7 oz. House Sirloin Applebee's Riblet Basket Oriental Chicken Salad Three Cheese Chicken Penne Chicken Tenders Basket Fiesta Lime Chicken

9. How many distinguishable permutations are there of the letters in each word?

a) COLORADO

b) MATHEMATICS

For questions 10 – 13, determine if each situation is a permutation or a combination, then answer the question.

10. Twenty students are semi-finalists for four $1000 scholarships. In how many different ways can the scholarships be awarded?

11. How many different ways can the starting 5 hitters on a baseball team line up?

12. Seven horses are in a race at Churchill Downs. You want to predict which horse will finish first, which second, and which third. How many different predictions are possible?

13. How many 6-letter "words" (not necessarily in any dictionary) can be formed from the letters in the word "turkey"?

Evaluate the following without a calculator.

14. $4!$

15. ${}_{10}P_4$

16. ${}_{10}C_4$

17. How can you calculate ${}_nC_r$ if you know ${}_nP_r$?

Precalculus
Worksheet 9.2

1. What is the 12^{th} row of Pascal's Triangle?

For questions 2 – 5, expand each expression using the Binomial Theorem or Pascal's Triangle.

2. $(2x+3)^6$

3. $(x-y)^3$

4. $(3x-2)^7$

5. $\left(\sqrt{x}+\sqrt{2}\right)^4$

6. Find the coefficient of the term containing y^5 in the expansion of the expression $(5x-4y)^8$.

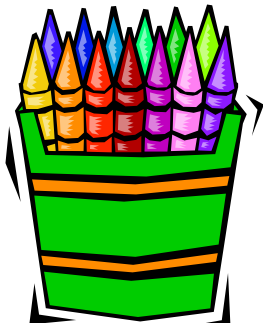
7. Find the coefficient of the term containing x^2 in the expansion of the expression $(4x+y)^{10}$.

Another one of those patterns from Pascal's Triangle …

8. Complete the following table by referring to Pascal's Triangle … including the last column.

Row #	0	1	2	3	4	5	6	7	…	*n*
Sum of Coefficients										

9. The box of crayons shown at the right has 13 different crayons.



a) How many ways can you grab 4 of them?

b) How many ways can you choose 9 to leave in the box?

c) Is there any difference between choosing 4 crayons to take and choosing 9 to leave?

d) Calculate the following pairs of combinations:

i) ${}_7C_2$ and ${}_7C_5$

ii) ${}_9C_3$ and ${}_9C_6$

iii) ${}_{10}C_2$ and ${}_{10}C_8$

e) What do you observe about each pair of combinations in part *d* and your answers for parts *a* and *b*?

f) Why is ${}_nC_r = {}_nC_{n-r}$?
[*Hint*: Use the combination formula to show ${}_nC_{n-r}$ is the same as ${}_nC_r$.]

An extension to 9.1

10. A Chinese restaurant will make a Pu-pu platter "to order" containing any one, two, or three selections from its appetizer menu. If the menu offers five different appetizers, how many different platters could be made?

An Activity To Do Together …

In calculus, we prove the "power rule" for derivatives using the binomial expansion formula and limits.

The power rule says $\lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = n\cdot x^{n-1}$

In other words, we want to show the limit as "*h* gets close to 0" of the fraction $\frac{(x+h)^n - x^n}{h}$ is equal to $n\cdot x^{n-1}$.

First thing we have to do is simplify the fraction $\frac{(x+h)^n - x^n}{h}$.

Step 1: Using the Binomial Theorem, what does the numerator of the fraction above simplify to?
[Your expression will look similar to the one on your notes … see the "Binomial Theorem" box …]

$(x+h)^n - x^n$ =

… Did you use the facts that ${}_nC_n = 1$ and ${}_nC_1 = n$?

Step 2: The expression $\frac{(x+h)^n - x^n}{h}$ means that we can take the simplified answer from step1 and divide every single term by *h*. Do this.

Step 3: In calculus, finding the "limit as *h* approaches 0" is done by plugging in *h* = 0 AFTER simplifying the expression. Therefore, we can find $\lim_{h\to 0}\frac{(x+h)^n - x^n}{h}$ by making *h* = 0 in the expression we found in step 2.

… Is your result what we wanted?

Pre Calculus
Worksheet 9.3 Day 1

1. Is this a valid probability function? If so, explain why. If not, change the question to make it valid.

Color	Brown	Red	Yellow	Green	Orange
Probability	0.3	0.2	0.2	0.1	.01

2. Suppose Jim draws a card from a standard deck of 52 cards. Suppose Jim draws a card, looks at it, and then **returns it to the deck** and draws again.

a) What is the probability that he picks a queen?

b) What is the probability that he picks a queen given that the card selected is known to be a face card?

c) What is the probability that exactly one of the two cards Jim draws is a queen?

d) What is the probability that at least one of the two cards is a queen?

e) What is the probability that Jim selects a queen or a ten?

f) What is the probability that he selects a queen or a heart? (HINT: Are these events mutually exclusive?)

3. Consider the situation where two dice of different colors are rolled. List the set of all possible outcomes, also known as the sample space. Then, use the sample space to find the probability of each event or events below.

a) P (doubles)

b) P (sum of dice = 7)

c) P (sum of dice < 10)

d) P (at least one die shows a 4)

e) P (doubles or sums over 7)

4. A bag contains 12 blue marbles and 16 green marbles. Two marbles are drawn at random, one after the other. One marble is drawn at random and not replaced. Then a second marble is drawn. What is the probability that the first marble is blue and the second one is green?

5. Floppy Jalopy Rent-a-Car has 30 cars available for rent: 15 SUVs, 8 midsize cars and 7 compact cars. If two cars are selected at random, what is the probability that both are compact cars?

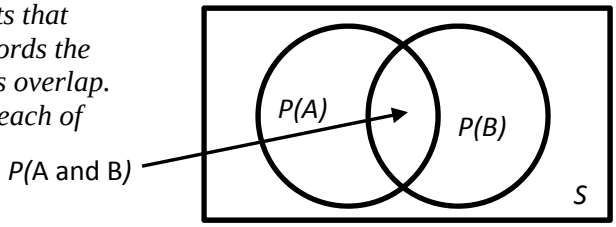
6. Suppose you select a card from a deck of cards, keep it, and then select another card. Find the following probabilities.

a) The first card is a heart and the second card is a club

b) The first card is a heart and the second card is a Queen

c) Only one of the two cards selected is an Ace

A Venn Diagram is a visual representation for two or more events that are not mutually exclusive in the same sample space. In other words the events CAN occur at the same time as indicated where the circles overlap. Remember to make your diagram complete with probabilities in each of the four regions on the diagram.



8. Suppose *A* and *B* are events in the sample space *S* where *P(A)* = 0.7, *P(B)* = 0.4 and *P(A* and *B)* = 0.2. **Draw a Venn diagram of this situation.** Then, find each probability.

a) Are these events mutually exclusive? How do you know?

b) Find the probability that *A* occurs, but *B* does not.

c) Find the probability that *B* occurs, but *A* does not.

d) Find the probability that neither *A* or *B* occur.

8. The probability of rain on Monday is 0.24, the probability of rain on Tuesday is 0.6 and the probability of rain on both days is 0.10. **Draw a <u>Venn diagram</u> of this situation**. Then, find each probability.

a) P(rains only Tuesday)

b) P(rains only Monday)

c) P(no rain either day)

9. Participants at a two-day conference could register for the 1st day, the 2nd day or both days. 66% of the participants registered for the 1st day and 72% of the participants registered for the 2nd day.
Draw a <u>Venn diagram</u> of this situation. Then answer the questions.

a) Are these events mutually exclusive? Explain how you know.

b) If there were 350 total participants, how many people attended the 2nd day only?

c) What is the probability that a randomly selected person attended both days?

Remember the tree diagram example from the video? See example 7 on page 725 of your textbook for additional help! Let's practice that strategy now…

For questions 10-13, create a tree diagram for the situation complete with probabilities! Use it to help you answer the questions that follow.

10. Suppose there is a 40% chance of snow tomorrow. If it snows, there is a 20% chance that the basketball game in Steamboat will still be played. If it doesn't snow, there is a 90% chance that the basketball game in Steamboat will still be played.

a) What is the probability that the basketball game will be played tomorrow?

b) What is the probability that the basketball game will be played, given that it did not snow?

11. A certain school has 40% of it's students on the honor roll. Of the students on the honor roll, 95% are going to college. Of the students not on the honor roll, 85% will go to college. Find the following probabilities if a student is selected at random.

a) P(student is on the honor roll and is NOT going to college)

b) P(student is going to college)

c) P(student is on the honor roll given that they are going to college)

12. Suppose that the probability of randomly selected person being a smoker is 0.25. Assume further that the probability of a smoker getting lung cancer is 0.32 and a person who does not smoke has a 0.05 probability of getting lung cancer. What is the probability that the individual is a smoker, given that he/she has lung cancer?

13. Using observations made of drivers arriving at a certain high school, a study reports that 68% of adults wear seat belts while driving. A high school student also in the car wears a seat belt 65% when the adult wears a seat belt, and 26% of the time when the adult does not wear a seat belt. What is the probability that the adult wears a seatbelt, given the student passenger is wearing a seatbelt?

To prepare for the 2nd half of 9.3, answer the following questions BEFORE you watch the video!

Warm Up for Binomial Probability: Suppose you and a friend are rolling a single six sided die …

1. What is the probability that your friend rolls a six exactly once in four rolls? Be careful to consider ALL scenarios…What if you roll the six first? What if you roll the six last? Etc.

2. How does this question change if we want to find the probability that the friend rolls a six exactly twice in four rolls?

Pre Calculus
Worksheet 9.3 Day 2

1. Suppose that 23% of all college students are married. Answer the following questions for a random sample of 8 college students.

a) What is the probability that 5 of the students are married?

b) What is the probability that 5 or more of the students are married?

2. A weighted coin comes up heads 70% of the time. If the coin is flipped 4 times, find the following probabilities.

a) P(exactly 3 heads)

b) P(at least 3 heads)

3. A baseball player gets a hit 33% of the time. If the player will get 11 at bats in the next couple of games, find the following probabilities.

a) P(exactly 3 hits)

b) P(no more than 3 hits)

4. Suppose you have a multiple choice test in Chemistry and you completely forgot to study. Worse yet, you really haven't paid attention at all, so you are going to have to guess on every single question. The test consists of 25 questions each having 4 choices.

a) What is the probability that you actually answer all 25 correct.

b) You would be the happiest student alive if you could score an 80% on the test. What is the probability that you score exactly 80%?

c) What is the probability that you score at least 80%?

5. In a random check on an assembly line, the probability of finding a defective item is 0.003. If 10 items are inspected, find the following probabilities:

a) P(none are defective)

b) P(exactly 1 is defective)

c) P(at least 1 is defective)

Your TI-83+ calculator will calculate binomial experiment probabilities using the functions *binompdf* (binominal probability distribution function) and *binomcdf* (binomial cumulative distribution function).
(Press 2nd ,VARS, then select A:binompdf or B:binomcdf.)

The *binompdf* function will calculate probabilities for an "exact" number of trials.
Use the following syntax: *binompdf* (number of trials, probability of success, number of successes)

The *binomcdf* function will calculate probabilities and add them together. If entered using the following syntax it sums the probability from 0 success to *k* successes:

binomcdf (number of trials, probability of success, *k* successes)

5. Go back and do question 3 using the functions of your calculator. Use *binompdf* to do part *a* and *binomcdf* to do part *b*. Did you get the same answers?

6. How could you use the *binomcdf* function to answer question 1b ?

Warm Up Questions for lesson 9.4:

1. Write the next three numbers for each pattern:

a) 3, 7, 11, … b) 25, 20, 15… c) 108, 54, 27, …

2. What is different about the last set of numbers in part c?

3. Think about this for question 1a and 1b…

How can you find the 21st number in the pattern WITHOUT finding all the numbers in the middle first? Would it help if you knew what the constant change between each number was? Do you know this information? How many times would you need to use this constant change amount?

Pre Calculus
Worksheet 9.4 Day 1 (Sequences)

1. A sequence is arithmetic if the _____ between any two consecutive terms is the same.

2. A sequence is geometric if the _____ between any two consecutive terms is the same.

3. Explain the difference between an explicit and a recursive formula.

4. Consider the sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$

   What is the 1st term? What is the 5th term? What is the *n*th term?

   What is the term before the *n*th term? What is the term after the *n*th term?

5. Consider the sequence defined by the formula $a_n = 5n - 7$.

   a) Is this sequence geometric or arithmetic? … Is the formula a recursive or explicit formula?

   b) Find $a_1, a_2, a_3, a_4,$ and a_5.

   c) Find the value of a_{50}.

6. Consider the sequence defined by the formula $\begin{cases} a_1 = 5 \\ a_n = a_{n-1} \cdot 3 \quad \text{if} \quad n \geq 2 \end{cases}$

   a) Is this sequence geometric or arithmetic? … Is the formula a recursive or explicit formula?

   b) Find $a_1, a_2, a_3, a_4,$ and a_5.

   c) Find the value of a_{50}.

For questions 7 – 10, determine whether each sequence is arithmetic or geometric. Then write both a recursive and an explicit formula for each sequence.

7. $\frac{3}{4}, \frac{3}{2}, 3, 6...$

8. 29, 22, 15, 8, …

9. 0.4, 0.04, 0.004, 0.0004, …

10. $2, \frac{6}{5}, \frac{18}{25}, \cdots$

11. The corner section of a football stadium has 15 seats in the first row and each successive row contains two additional seats. How many seats are in the 40^{th} row?

12. The Louvre pyramid in Paris, France, is built of glass panes. There are 4 panes in the top row, and each additional row has 4 more panes than the previous row. If the pyramid is made of 18 rows, how many panes will be in the 18^{th} row?

13. The third and sixth terms of a geometric sequence are –75 and –9375 respectively. Find the first term, the common ratio and an explicit formula.

14. The fifth and twenty-third terms of an arithmetic sequence are –7 and –97 respectively. Find the first term, the common difference, and an explicit formula.

15. In an arithmetic sequence, $a_4 = -8$ and $a_7 = 4$. Find the first term and write a recursive rule.

16. In a geometric sequence, $a_{12} = 14,336$ and $a_6 = 224$, find a_1 and write a recursive rule.

17. One hundred loaves of bread are to be divided among five people so that the amounts that they receive form an arithmetic sequence. The first two together receive one-seventh of what the last three receive. How many does each receive?

Need more practice? … See pages 746 - 747 #1 – 22 .

Pre Calculus
Worksheet 9.4 Day 2 (Series)

1. Describe the difference between a sequence and a series.

2. Suppose $a_1 = 2$, $a_2 = 5$, $a_3 = 9$, $a_4 = 12$, and $a_5 = 21$. Complete the following:

a) $\sum_{i=1}^{3} a_i$ b) $\sum_{i=2}^{5} (a_i + 2)$ c) $\sum_{i=1}^{3} (3a_i)$

3. Why is $\sum_{n=1}^{\infty} 5(2)^n$ the same as $\sum_{n=2}^{\infty} 5(2)^{n-1}$?

When writing a series using summation notation always use the EXPLICIT FORMULA.

For questions 4 – 7, write each series using summation notation. You're not finding the sum yet … just rewriting the series using summation notation.

4. $0.5 + 1.6 + 2.7 + 3.8 + \cdots + 14.8$

5. $4 - 8 + 16 - 32 + \cdots - 32768$

6. $9 + 3 + 1 + \cdots$

7. $4.8 + 7.2 + 10.8 + 16.2 + \cdots$

8. Three of the four series above actually have a sum that can be calculated. Which one cannot? Why?

Btw … what do we call a series where a sum can actually be calculated?

9. Now it's time to find the sums … go back to questions 4 – 7 and calculate the sum of each series if possible.

For 10 - 12, find each sum, if it exists. If it does not exist, explain why not.

10. $\sum_{n=1}^{7}(2n+5)$

11. $\sum_{n=1}^{8}3(2)^{n-1}$

12. $18+12+8+\frac{16}{3}+...$

13. 18 + 11 + 4 + …+ (–31)

14. $\sum_{n=1}^{\infty}2\left(\frac{3}{4}\right)^{n}$

15. $-6+5-\frac{25}{6}+\frac{125}{36}-\cdots$

16. How many rows are in a corner section of a stadium containing 2040 seats if the first row has 10 seats and each successive row has 4 additional seats?

17. A ball is dropped from a height of 16 feet. Each time the ball drops *h* feet, it rebounds 0.81*h* feet. It can be shown (I decided not to make you do this in class today … aren't I nice!) that the total elapsed time that ball bounces before it comes to rest is given by the equation $t = 1 + 2\sum_{n=1}^{\infty} 0.9^n$. Find this total time.

18. You have decided to accept a strange job (don't worry … it's something you enjoy doing). What makes it strange is the way you will get paid. Your boss has agreed to pay you \$0.01 for the first day, \$0.02 for the second day, \$0.04 for the third day, etc.

a) How much will you earn on the 7^{th} day?

b) How much will you earn on the 30^{th} day?

c) What is the total amount of money you will earn in 30 days?

19. Michael is a chocoholic. One New Year's Day, he ate one piece of chocolate. On the next day, he ate 2 pieces. One each subsequent day, he ate one additional piece of candy.

a) How many pieces of candy did he eat on the last day of January?

b) How many pieces did he eat during the month of January? (and do you think he'll be alive in February?) ☺

20. Hopefully by now, you know that the sum of an arithmetic series can be found using the formula $\frac{n}{2}(a_1 + a_n)$.

a) What is the explicit formula for the *n*th term in an arithmetic sequence? $a_n =$ _____.

b) Substitute this value into the formula for the sum of an arithmetic series.

This formula is also used to find the sum of an arithmetic series, if you don't know (or just don't want to find) the last term in the series.

All repeating decimals can be written as a fraction. Geometric series can be used to accomplish this.

Example: Express the rational number $0.\overline{139}$ as a fraction.

First: Write the repeating parts as a geometric series …

0.139 + 0.000139 + 0.000000139 + 0. 000000000139 + ⋯

Second: The sum of this infinite geometric series is the fraction we are looking for, so find a_1 and r.

$a_1 = 0.139$ … and … $r = 0.001$

Third: Sum = $\frac{0.139}{1-.001} = \frac{0.139}{.999} = \frac{139}{999}$.

Therefore, $0.\overline{139} = \frac{139}{999}$.

21. Rewrite each of the following repeating decimals as a fraction.

a) 0.1414141414...

b) $5.\overline{297}$

Self –test … without looking at your notes … how many of the following formulas do you know ..
Hint … you will have a formula quiz next class

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1. Recursive Formula for an Arithmetic Sequence:

2. Explicit Formula for an Arithmetic Sequence:

3. Recursive Formula for a Geometric Sequence:

4. Explicit Formula for a Geometric Sequence:

5. Formula for the Sum of a Finite Arithmetic Sequence:

6. Formula for the Sum of a Finite Geometric Sequence:

7. Formula for the Sum of an Infinite Geometric Sequence:

   When does the formula from #7 actually work?