P.1 REAL NUMBERS

Learning Targets for P1

1. Describe an interval on the number line using inequalities
2. Describe an interval on the number line using interval notation (closed vs. open)
3. Switch between interval notation and inequality notation
4. Simplify exponential expressions
5. Identify Algebraic Properties (Commutative, Associative, Distributive, Identity, Inverse)

Inequalities and Intervals

When you first graphed inequalities you used an open circle or a solid dot. Interval notation is another way to describe intervals instead of using an inequality. The ends of an interval are either OPEN or CLOSED.

When using interval notation use ____________________ instead of open circles, and use ______________________ instead of solid dots.

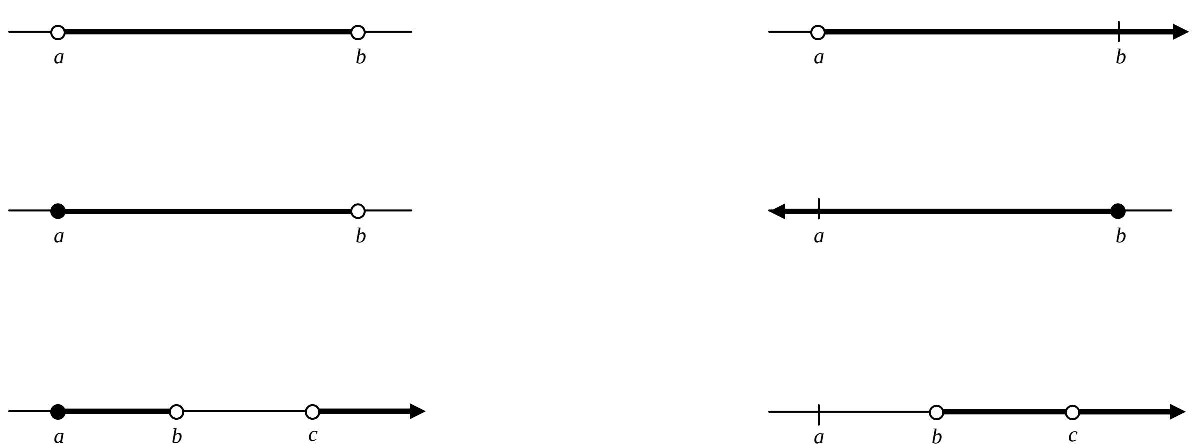
Example 1: Graph the following on a number line and write your answer using interval notation.

a) $5 < x$ b) $x \leq 2$ c) $7 < x \leq 12$

Example 2: Graph the following and write the corresponding inequality.

a) $(3, 5]$ b) $[-12, \infty)$ c) $(-\infty, 7)$

Example 3: Use interval notation to describe each interval on the *x* – axes below.



Simplifying Expressions with Exponents

Properties of Exponents

1. $u^m u^n =$
2. $\frac{u^m}{u^n} =$
3. $u^0 =$
4. $u^{-m} =$
5. $(uv)^m =$
6. $(u^m)^n =$
7. $\left(\frac{u}{v}\right)^m =$

Example 4: Simplify each of the following expressions.

a) $2x^{-1}$

b) $\left(\frac{3}{xy}\right)^{-2}$

c) $\left(\frac{3a^2b}{2a^3b}\right)\left(\frac{4b^3}{a^4b^2}\right)$

d) $\frac{(x^{-2}y^3)^{-2}}{(y^5x^{-2})^{-1}}$

Properties of Algebra

Commutative Property

Associative Property

Inverse Property

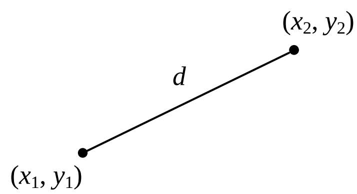
Identity Property

Distributive Property

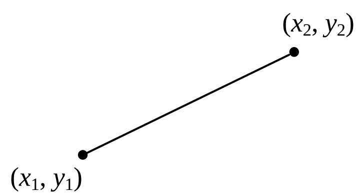
P.2 CARTESIAN COORDINATE SYSTEM

Learning Targets for P.2
1. Know and be able to use the distance formula
2. Know and be able to use the midpoint formula
3. Be able to identify a given equation as a circle
4. Be able to write the equation of a circle centered at (h, k) with a radius of r

Example 1: The distance formula is derived directly from the Pythagorean Theorem. Create a right triangle with the segment below, and solve for d.



Example 2: To find the midpoint of a segment simply find the average x and y-values of the endpoints.

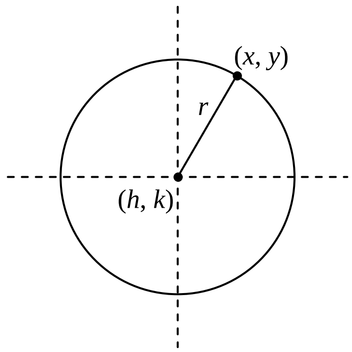


Example 3: Let the endpoints of a segment be (–2, 8) and (5, 7).

a) Find the length of the segment.

b) Find the midpoint of the segment.

Example 4: An equation of a circle whose center is at (h, k) can be written using the Pythagorean Theorem as well.



Example 5: Write an equation of the circle with radius 4 and whose center is at (0, –3).

Example 6: Sketch a graph of the equation $(x+1)^2 + (y-4)^2 = 9$.

P.3 LINEAR EQUATIONS AND INEQUALITIES

Learning Targets for P.3

1. Solve multiple step linear equations
2. Solve multiple step linear inequalities
3. Solve equations with fractions

Solving Linear Equations

Example 1: Solve each of the following equations..

a) $2(3 - 4b) - 5(2b + 3) = b - 17$

b) $\frac{x+3}{4} - \frac{x-1}{6} = 1$

c) $\frac{x-5}{15} + 4 = \frac{2x+1}{25}$

Solving Linear Inequalities

Example 2: Solve each inequality, graph the solution on a number line, and write the answer in interval notation.

a) $\frac{2y-3}{2} + \frac{3y-1}{5} > y - 1$

b) $\frac{1}{4}(x-4) - x \geq \frac{5}{2}(3-x)$

Example 3: Solve the following inequality and graph the solution on a number line:

$$-2 \leq 3x + 4 < 5$$

P.4 LINES IN THE PLANE

Learning Targets for P.4

1. Calculate Average Rate of Change between 2 points
2. Write and graph an equation of a line in point-slope form
3. Write and graph an equation of a line in slope-intercept form
4. Write and graph an equation of a line in general form
5. Write and graph equations of horizontal and vertical lines
6. Understand how the slopes of parallel lines are related
7. Understand how the slopes of perpendicular lines are related

Slope

The slope of a non-vertical line is given by $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

A vertical line has ________________________, and a horizontal line has ________________________ .

Parallel Lines have slopes that are __________________ .

Perpendicular Lines have slopes that are ______________________________________.

IMPORTANT ♬: You will be best served in this class if you think of slope as a ________________________________.

Equations of a Line

The first equation of a line you used in algebra was probably the *slope – intercept form*: ___________________________

The slope is ______, and the *y*-intercept is ________.

In precalculus, it is actually easier to write the equation of a line in *point – slope form*: ____________________________

The point is ________, and the slope is __________.

♬: To write an equation of a line, all you need is a ___________________ and the _________________.

Another format used to write the equation of a line is called *standard (general) form*: ____________________________

Example 1: Which of the equations above has "*y* written as a function of *x*" ?

Example 2: The point-slope form is written as ______________________________ if you want "*y* written as a function of *x*"

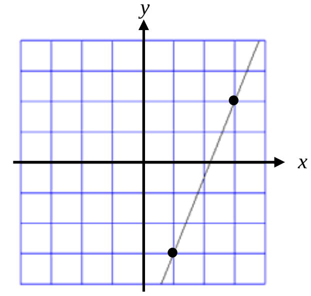
Example 3: For each of the following, write the equation of the line with the given information in point-slope form.

a) Point (–2, 6); Slope = –1

b) Point (1, –3); Slope = $\frac{5}{6}$

c) Points (12, 0) and (6, 3) are on the line.

d)



Example 4: Find the slope-intercept form of the line passing through (–2, 4) and having the following characteristics:

a) Slope of $\frac{7}{16}$

b) Parallel to the line $5x - 3y = 3$

c) Passing through the origin

d) Parallel to the y – axis.

Example 5: Find the general form of the line passing through (1, 3) and having the following characteristics:

a) Slope of $-\frac{2}{3}$

b) Perpendicular to the line $x + 2y = 0$

c) Passing through the point (2, 4)

d) Parallel to the x – axis.

FACTORING

Learning Targets for Factoring
1. Factor out a GCF
2. Factor difference of squares
3. Factor a quadratic expression of the form $ax^2 + bx + c$
4. Completely Factor an expression (including grouping)

<u>*Greatest Common Factor* (GCF)</u>

The first step in factoring is to factor out a GCF. We did this in P.1.

Example 1: Factor out the GCF from each expression.

a) $3x^2 + 6x$

b) $5x^4 - 7x^3 + 2x^2$

<u>*Factoring Quadratic Expressions of the Form $ax^2 + bx + c$*</u>

Example 2: Factor each expression completely.

a) $3x^2 - 4x - 7$

b) $6x^2 - 2x - 8$

c) $6x^2 - 19x + 15$

<u>*Difference of Two Perfect Squares*</u>

Example 3: Factor each expression.

a) $a^2 - b^2$

b) $9x^2 - 25y^2$

c) $36x^2 + 49$

Example 4: <u>***Completely***</u> factor each expression.

a) $4x^4 + 24x^3 + 32x^2$.

b) $3(2a-3)^2 + 17(2a-3) + 10$

P.5 SOLVING EQUATIONS GRAPHICALLY, NUMERICALLY, AND ALGEBRAICALLY

Learning Targets for P.5

1. Solve equations graphically with a calculator (2 ways)
2. Solve quadratic equations algebraically: x^2 but no x, x^2 and x but no c, x^2 and x and c, undo $(\ldots)^2$
3. Complete the square
4. Solve Absolute value equations

Being able to solve equations in multiple ways allows you to answer a question using the easiest method possible. Another advantage of having multiple ways to solve an equation is that it gives you an opportunity to check your work.

The three methods are obvious from the title of the section … Graphically, Numerically, and Algebraically.

Solve an Equation by Graphing

There are two keys to solving an equation by graphing.

1. Understanding WHERE the solution(s) is(are).
2. Finding those solutions using your calculator (or a hand drawn graph).

Solutions are located in _________ possible places, either the _____________________or _________________________.

Example 1: Solve the equation $2x^2 - 5x = 3$ by graphing the left side of the equation and the right side of the equation.

a) Where is(are) the solution(s) to this equation located?

b) What is the solution to this equation?

Example 2: Solve the equation $2x^2 - 5x = 3$ by setting one side equal to 0, and graphing the other side.
(This is equivalent to finding the ZEROS of a function)

a) Where is(are) the solution(s) to this equation located?

b) What is the solution to this equation?

Solve an Equation Numerically

Typically, you will be given a table of values when asked to solve an equation numerically. You could also create the table yourself.

Example 3: Use the table to approximate the solution to the equation $0 = -x^3 + x + 1$.

x	y
1.1	0.769
1.2	0.472
1.3	0.103
1.4	– 0.344
1.5	– 0.875
1.6	– 1.496
1.7	– 2.213

Solve an Equation Algebraically

This is the method you have spent the most time using in math classes so far. We focused on linear equations earlier in chapter P. In this section we are going to focus on various methods of solving *quadratic* equations. Polynomials, rational equations, exponential, logarithmic, and trigonometric equations will be done in later sections throughout the year.

Method 1: Square Root Both Sides

Example 4: Solve by extracting square roots.

a) $3x^2 - 7 = 14$

b) $2(2y+3)^2 - 9 = 23$

Method 2: Complete the Square

Example 5: Solve by completing the square.

a) $x^2 + 6x = 7$

b) $2x^2 + 5x - 12 = 0$

Method 3: Quadratic Formula

Example 6: Solve using the quadratic formula.

a) $2x^2 - x - 3 = 0$

b) $3x^2 - 6x - 7 = x^2 + 3x - x(x+1) + 3$

Method 4: Factoring

Example 7: Solve by factoring.

a) $7x^2 = 3x$

b) $4x^2 + 3 = 8x$

Definition of Absolute Value

Absolute Value is best viewed as a distance. When finding the absolute value of a single number, you are finding the ________________________ that number is from ________________. We will talk about the algebraic definition more when we graph functions in the next chapter.

Example 8: Solve for all values of *x*: $|x| = 7$

Example 9: Consider the equation $2|3x+5|-8=19$.

a) Solve the equation algebraically.

b) Solve the equation graphically.

P.6 SOLVING INEQUALITIES ALGEBRAICALLY AND GRAPHICALLY

Learning Targets for P.6

1. Solve Absolute Value Inequalities using correct notation and vocabulary
2. Solve quadratic or cubic inequalities by finding zeros on a graph
3. Apply solving inequalities to context including but not limited to projectile motion

Rules to Remember When Solving Absolute Value Inequalities

Let *u* be an algebraic expression in *x* and let *a* be a real number greater than 0.

1. $|u| < a$ if and only if $-a < u < a$
2. $|u| > a$ if and only if $u < -a$ or $u > a$

Informally, what these means is if you are represented by 👤, then |👤| represents your distance from zero.

Example 1: If |👤| > 5, use a number line to represent where YOU are allowed to be.

Example 2: If |👤| < 5, use a number line to represent where YOU are allowed to be.

Example 3: Solve each inequality algebraically. Graph your solution & write your solution in interval notation.

a) $|2x-1| > 35$

b) $13-4|3-4x| \le 9$

<u>Solving Inequalities Without Absolute Values … USING SIGN CHARTS!!!!!</u>

We will spend much more time with this later on in the year, but for now, a quick introduction.

Example 6: Solve $x^3 + x^2 - 6x < 0$

a) Graphically

b) Algebraically using the ZEROS and a SIGN CHART

Example 7: [Calculator Allowed] The height, *h*, in feet of a projectile *t* seconds after is has been launched vertically from an initial height of h_0 feet above the ground with an initial velocity of v_0 feet/second is given by the formula

$$h(t) = -16t^2 + v_0 t + h_0$$

a) If an object is launched with an initial velocity of 256 ft/sec from the ground, write the equation modeling the height of the object.

b) When does the object hit the ground?

c) When does the object reach it's maximum height?

d) What is the maximum height of the object?

e) When will this object be at least 768 feet above the ground?

2.8 SOLVING RATIONAL EQUATIONS

Learning Targets for 2.8

1. Be able to find the LCD of 3 polynomials
2. Solve a Rational Equation by clearing the fractions
3. Understand when a solution to a rational equation is extraneous and identify them when found

Example 1: When we solved equations involving fractions, we eliminated the fractions by

Example 2: Find the least common denominator of the following pairs of fractions:

a) $\frac{1}{3t}$ and $\frac{1}{5t^2}$

b) $\frac{x}{8}$ and $\frac{3x}{2}$

c) $\frac{4}{3h^2}$ and $\frac{2}{h^3}$

d) $\frac{4}{2z^2+2z}$ and $\frac{5z}{z+1}$

e) $\frac{4}{y+2}$ and $\frac{3+y}{y-1}$

f) $\frac{1}{k+2}$ and $\frac{3k}{k^2-4}$

g) $\frac{7x-6}{x^2-7x-8}$ and $\frac{3x}{2x^2-19x+24}$

Example 3: Solve the following equations.

a) $\frac{a}{3}+\frac{a}{5}=4$

b) $\frac{5}{4x}+\frac{2}{3}=\frac{1}{x}$

Whenever you solve rational equations (or square root equations or logarithmic equations) you create the possibility of extraneous solutions. You are NOT allowed to have a value of 0 in the denominator of a fraction, so IF your algebra leads you to a solution that would give you a value of 0 in the denominator of the original equation, that answer is extraneous.

Example 4: Solve the following equations. Check for extraneous solutions.

a) $\dfrac{3}{x+2}=\dfrac{4}{x-3}$

b) $\dfrac{5}{2x-2}=\dfrac{15}{x^2-1}$

c) $\dfrac{2}{x-3}-\dfrac{4}{x+3}=\dfrac{8}{x^2-9}$

d) $\dfrac{n}{n^2+2n}+\dfrac{1}{n}=\dfrac{3}{n+2}$

e) $\dfrac{3x}{x+5}+\dfrac{1}{x-2}=\dfrac{7}{x^2+3x-10}$

f) $\dfrac{2}{x+3}-\dfrac{3}{4-x}=\dfrac{2x-2}{x^2-x-12}$