2.5 COMPLEX NUMBERS

Learning Targets:

1. Understand that $\sqrt{-1}$ is an imaginary number denoted by the letter *i*.
2. Evaluate the square root of negative numbers.
3. Understand that complex numbers look like *a* + *bi*, where *a* is real and *bi* is imaginary.
4. Know how to Add and Subtract Complex Numbers.
5. Know how to Multiply Complex Numbers and simplify powers of *i*.
6. Know how to find a complex conjugate.
7. Know how to use the complex conjugate to Divide complex numbers.
8. Know how to solve a quadratic equation using a domain of complex numbers.

Example 1: Solve $x^2 + 1 = 0$.

While the equation in example 1 may not have a real numbered solution it does not mean that there isn't a solution, nor does it mean that the solution itself has no meaning. The solution to example 1 will involve an imaginary number *i*.

We will define $\sqrt{-1}$ = ___________.

Just because we refer to these solutions as imaginary does not mean that the solutions are meaningless. Fields such as quantum mechanics and electromagnetism depend on the mathematics of imaginary numbers. When engineers design airplane wings or cell-phone towers, imaginary numbers are vital to their calculations. Applications abound in electrical engineering, vibration engineering, polymer science, navigation, and more. Applications abound in the real world, touching our lives via design of popular features such as the vibrating ringer in our cell phones or the bass boosters in our MP3 players. More heavy duty applications include the design of missile guidance systems. While we are not ready to dive into these topics yet, we are ready to learn to simplify the square root of negative numbers correctly.

Example 2: Evaluate each square root:

a) $\sqrt{-9}$

b) $\sqrt{-50}$

Complex Numbers are made up of real and imaginary parts. They look like ____________________. Whenever you add, subtract, multiply, or divide complex numbers, your answer will be in the form of a complex number as well.

Example 3: Add or Subtract the following complex numbers:

a) $(5+3i)+(-7-8i)$

b) $(5+3i)-(-7-8i)$

Multiplying Complex numbers starts off just as easily. However, if you ever end up with a power of *i* that is greater than 1, you must learn to simplify so that your answers are of the form *a* + *bi*.

Example 4: Simplify the following: i^2

Example 5: Multiply the following complex numbers:

a) $(5+3i)(-7-8i)$ b) $(6+2i)^2$ c) $(3+5i)(3-5i)$

<u>*Complex Conjugates*</u>: The complex conjugate of *a* + *bi* is _______________.
Look back at example 5*c* … what happened when you multiplied complex conjugates together?

Remember, when you add, subtract, multiply, AND divide complex numbers, the answer is also a complex number. Division of complex numbers uses the complex conjugate to eliminate the imaginary part of the denominator. Subtraction was "*adding the opposite*" … dividing fractions was "*multiplying by the reciprocal*" … so it is with complex numbers …

Dividing complex numbers is going to be done by multiplying the numerator and the denominator by the __________________________ of the denominator.

Example 6: Divide $\dfrac{5+8i}{-7+2i}$ … be sure to write your answer as a complex number.

Up until now, the solutions to an equation could be found by setting the equation equal to zero, graphing the equation and finding where the function crossed the *x*-axis. However, if the solutions are imaginary, the function will NOT cross the *x*-axis. A quadratic function with NO real solutions is called "irreducible".

Example 7: Solve the following quadratic function if the domain is the set of complex numbers: $y=-3x^2+6x-7$
(Graph it on your calculator to determine how many real solutions it has)

2.6 COMPLEX ZEROS AND THE FUNDAMENTAL THEOREM OF ALGEBRA

Learning Targets:

1. Know the number of zeros a polynomial can have.
2. Understand that complex zeros come in pairs.
3. Given the zeros (real and complex) of a polynomial, find the standard form of the polynomial.
4. Given a polynomial, find all the real and complex zeros.
5. Use the zeros of a polynomial to write a polynomial as a product of linear and irreducible quadratic factors.

First … you should become familiar with the following theorems and concepts …

Fundamental Theorem of Algebra (NEW): A polynomial function of degree $n > 0$ has n complex zeros. Some of these zeros may be repeated.

Multiplicity (Ch 2A): The number of times a zero is repeated.

Factor Theorem (Ch 2A): If c is a zero, then $(x - c)$ is a factor and vice-versa.

Complex Conjugate Zeros (NEW): If $x = a + bi$ is a zero of a polynomial, then $x = a - bi$ … the conjugate … is also a zero.

Factors of a Polynomial with Real Coefficients: Every polynomial function with real coefficients can be written as a product of linear factors and irreducible quadratic factors*, each with real coefficients.

*An irreducible quadratic factor is a quadratic with NO real solutions (i.e. 2 complex solutions).

Example 1: Suppose you have a polynomial function with zeros $x = -2$ and $x = 3 + i$.

a) What is the minimum degree of this polynomial?

b) Write a polynomial function in factored form with these given zeros.

c) Write a polynomial function in standard form with these given zeros.

Now that we have gone from the zeros to the equation of the polynomial in standard form, let's revisit using the standard form of a polynomial to find the zeros as they relate to multiplicity and complex zeros.

Example 2: Given the function $y = x^4 - 8x^3 + 49x^2 - 132x + 116$.

a) Use your graphing calculator to find ALL the zeros.

b) Write the function from part (a) as a product of linear and irreducible quadratic factors.

2.7 Graphs of Rational Functions

Learning Targets:

1. Graph a rational function by hand including all of the following (if they exist)
   - i) End Behavior (including Horizontal Asymptotes and Slant Asymptotes)
   - ii) Vertical Asymptotes (distinguish between holes and vertical asymptotes)
   - iii) *x*-intercept(s)
   - iv) Additional points in each "region" to determine shape of graph (including the *y*-intercept)

Rational Function: $y = \dfrac{f(x)}{g(x)}$

Example 1: Use the equation below to answer the following questions: $h(x) = \dfrac{3x-9}{2x^2-11x+15}$

a) Find the end behavior. Include HA or slant asymptotes.

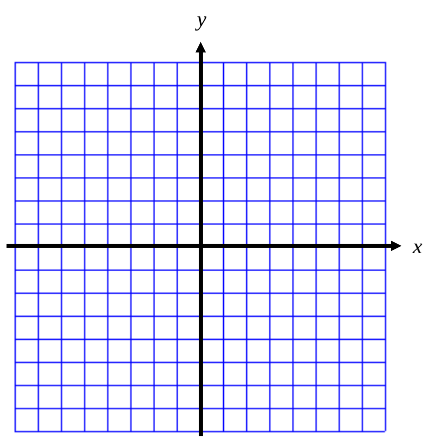
b) Find any vertical asymptotes.
Distinguish between holes and vertical asymptotes.

c) Find the *x*-intercept(s).

d) Find the *y*-intercept.

Example 2: Graph the equation from example 1

… include everything you found in example 1 AND additional points to determine the shape of the graph.

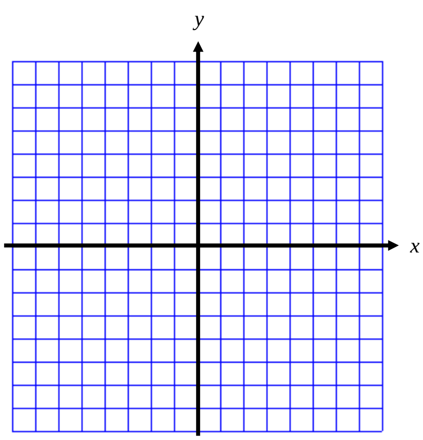


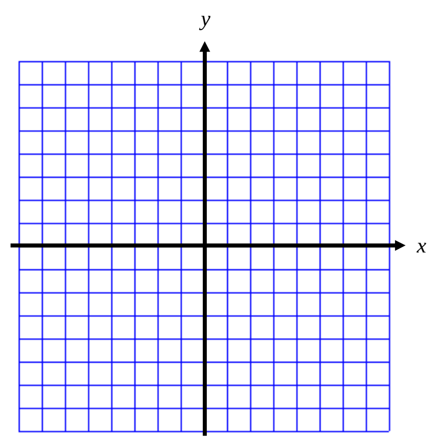
For each function find the following (if they exist).

a. End Behavior including the equations of horizontal and slant asymptotes
b. Vertical Asymptote(s). Distinguish between holes and vertical asymptotes.
c. *x*- intercept(s)
d. *y*-intercept and additional points in each region to determine the shape.
e. Graph without a calculator.

Example 3: $g(x)=\dfrac{x^2+6x+8}{x+1}$

Example 4: $k(x)=\dfrac{3x+1}{x^2+2x-3}$





2.9 Solving Inequalities in One Variable

Learning Targets:
1. Make a sign chart of a function using the values of *x* where the function is zero or undefined
2. Use a sign chart (or a graph) to determine the intervals where a function is positive or negative.
3. Understand how … (… and … [… relate to … < … and … $\leq$ when solving inequalities.

Polynomial functions can be positive, negative or zero.

- To solve $f(x) > 0$ …
- To solve $f(x) < 0$ …

Example 1: Sketch the following function by hand: $f(x) = (x-2)^2(x+3)(x-5)$
(Use what you know about zeros of the function and the end behavior)

Example 2: Use your graph to answer the following inequalities:

a) Find the values of *x* so that $(x-2)^2(x+3)(x-5) > 0$.

b) Find the values of *x* so that $(x-2)^2(x+3)(x-5) \geq 0$.

c) Find the values of *x* so that $(x-2)^2(x+3)(x-5) < 0$.

d) Find the values of *x* so that $(x-2)^2(x+3)(x-5) \leq 0$.

Sign charts are simply number lines with (pos) or (neg) signs on them to represent whether or not the function's *y*-values are positive or negative. For polynomial functions, the critical numbers for the sign chart are the zeros of the function.

Example 3: Make a "sign chart" for the function $f(x) = (x-1)^2(x-3)(x+5)$

Example 4: Solve the following inequality: $(x-1)^2(x-3)(x+5) < 0$
(Use your sign chart from example 3)

For the remainder of this lesson, we will deal with rational functions (a.k.a. fraction of polynomials). Rational functions can be positive, negative, zero or undefined.

♬ : A fraction equals 0 when the numerator equals 0.
A fraction is undefined when the denominator equals 0. **Always use ( ) where the fraction is undefined.

For this reason, we need to include BOTH of these values in our sign charts.

Example 5: Make a sign chart for $g(x)=\dfrac{3x+1}{(x-1)(x+3)}$. Indicate which values are zeros and which values are undefined.

Example 6: Answer the following inequalities using the sign chart from example 5:

a) Find the values of *x* so that $\dfrac{3x+1}{(x-1)(x+3)}<0$.

b) Find the values of *x* so that $\dfrac{3x+1}{(x-1)(x+3)}\le 0$.

c) Find the values of *x* so that $\dfrac{3x+1}{(x-1)(x+3)}>0$.

d) Find the values of *x* so that $\dfrac{3x+1}{(x-1)(x+3)}\ge 0$

In order to make the sign chart, one side of the inequality MUST be written as a single fraction.

Example 7: Solve the following inequality: $\dfrac{1}{x-1}+\dfrac{2}{x+3}\le 0$