2.1 LINEAR AND QUADRATIC FUNCTIONS AND MODELING

Learning Targets:
1. Understand what a polynomial function looks like.
2. Understand that Average Rate of Change implies slope between two points.
3. Model Linear functions using function notation and the regression capabilities of your calculator.
4. Find the vertex and graph a quadratic function in standard, intercept, and vertex forms.
5. Model Quadratic functions in vertex form.
6. Use the projectile motion model to find the highest point a projectile reaches, and when it reaches that point.

Chapter 2 deals with polynomial functions and you will learn about various aspects of these types of functions. Our first goal is to define polynomial functions and then to identify which functions are polynomials.

Definition of a Polynomial Function

A function $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$,

where n = ________________________________ and n is an ____________________________.

Example 1: Which of the following functions are polynomial functions? For those that are, state the degree and leading coefficient. For those that are not polynomials state why not.

$f(x) = 3x - 5x^2$ $h(x) = \sqrt{x^2 - 4}$ $s(t) = 2t^4 + 5t^3$ $g(x) = 6$ $k(x) = 4x^{\frac{2}{3}} - 5$

For the remainder of this section we will deal only with functions whose degree is less than or equal to 2.

Polynomials of degree 0 are called ____________________.

Polynomials of degree 1 are called ____________________.

Polynomials of degree 2 are called ____________________.

Modeling Linear Functions

Example 2: Using function notation, if $f(a) = b$, then the function contains the point _____________ .

Example 3: The **average rate of change** of a function between two points $(a, f(a))$ and $(b, f(b))$ is given by

Example 4: Write the equation of the linear function, f, if you know $f(2) = -7$ and $f(-1) = 5$.

Example 5: The table below shows the relationship between the number of Calories and the number of grams of fat in 9 different hamburgers from various fast-food restaurants.

Calories	720	530	510	500	305	410	440	320	598
Fat (g)	46	30	27	26	13	20	25	13	26

a) Find the linear regression model for this data.

b) In the context of this problem, what does the slope mean?

Modeling Quadratic Functions

Quadratic Functions are going to be given to you in the following forms:

$$f(x) = ax^2 + bx + c \qquad\qquad f(x) = a(x-h)^2 + k$$

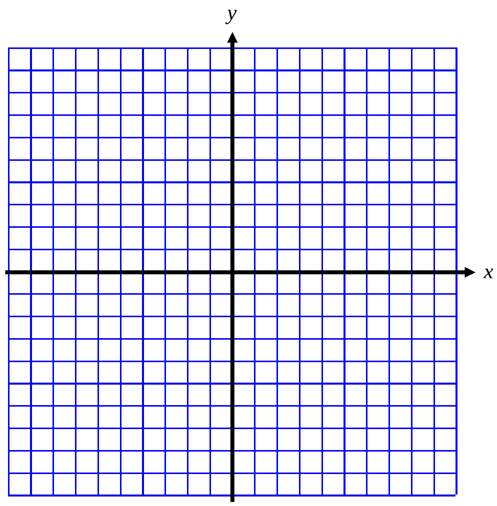
You need to be able to graph each form. The most important point will be the vertex.

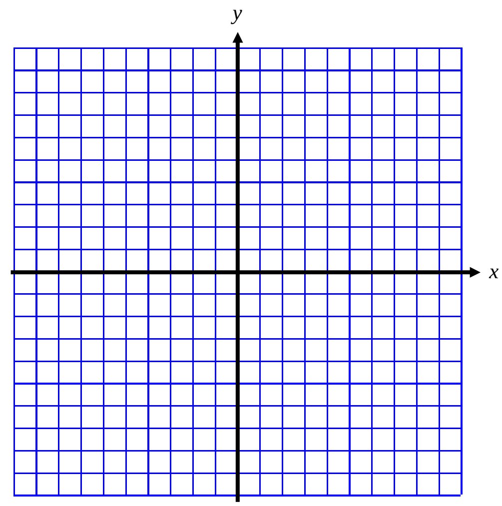
Example 6: Find the vertex of the quadratic function $f(x) = 2x^2 - 6x + 11$.

Example 7: Find the vertex of the quadratic function $f(x) = 3(x+5)^2 - 2$.

Once you find the vertex of a parabola, you can graph the remaining points by following the pattern below

Example 8: Graph the two quadratic functions from the previous examples.





Example 9: Find the equation of a quadratic function of the form $f(x) = a(x-h)^2 + k$ that passes through (–4, 15) and has it's vertex at (–2, 3).

Example 10: Change the quadratic function you wrote in the last example into the form $f(x) = ax^2 + bx + c$.

Ignoring air resistance, the height, *s*, of a free-falling object after *t* seconds can be modeled by the quadratic function

$s(t) = -16t^2 + v_0 t + s_0$, if *s* is in feet … or … $s(t) = -4.9t^2 + v_0 t + s_0$, if *s* is in in meters

Where v_0 = ____, and

s_0 = ____.

Example 11: At the Bakersville Fourth of July celebration, fireworks are shot by remote control into the air from a pit that is 10 feet below the earth's surface.

a) Find an equation that models the height of an aerial bomb *t* seconds after it is shot upwards with an initial velocity of 80 ft/sec.

b) Find the vertex of the quadratic function.

c) What is the maximum height above ground level that the aerial bomb will reach?

d) How many seconds after it was launched will it take to reach that height?

2.2 POWER FUNCTIONS WITH MODELING

Learning Targets:

1. Identify a power functions.
2. Model power functions using the regression capabilities of your calculator.
3. Understand the difference between “direct variation” and “inverse variation”.
4. Use power functions to solve word problems.

In the last section we studied linear functions and quadratic functions. In this section we move to *power functions*.

A power function looks like ____________________,

where a = ____, and k = ____.

We say that $f(x)$ ______________ as the a^{th} power of x, or that $f(x)$ is ____________________________ the a^{th} power of x.

If a is ______________________ we say that $f(x)$ varies ______________________ with the a^{th} power of x.

If a is ______________________ we say that $f(x)$ varies ______________________ with the a^{th} power of x.

Example 1: Determine if the function is a power function. For those that are not, explain why not.

a) $f(x) = -3x^4$ b) $f(x) = \sqrt[3]{8x^5}$ c) $g(x) = 7 \cdot 2^x$ d) $h(x) = 2x^{-5}$

Example 2: The volume V of a sphere varies directly as the cube of the radius r. When the radius of a sphere is 6 cm, the volume is 904.779 cm^3. What is the radius of a sphere whose volume is 268.083 cm^3?

Example 3: The force of gravity F acting on an object is inversely proportional to the square of the distance d from the object to the center of the earth. Write an equation that models this situation.

Example 4: Velma and Reggie gathered the data in the table below using a 100-watt light bulb and a Calculator-Based Laboratory(CBL) with a light-intensity probe.

a) Use your calculator to find the power regression model of the data.

b) Describe the relationship between the intensity and distance modeled with the equation in part *a*.

c) Use the regression model from part *a* to predict the intensity of an object 2.75 meters away.

Light Intensity Data for a 100-W Light Bulb

Distance(m)	Intensity(W/m^2)
1.0	7.95
1.5	3.53
2.0	2.01
2.5	1.27
3.0	0.90

2.3 POLYNOMIAL FUNCTIONS OF HIGHER DEGREE WITH MODELING

Learning Targets:

1. Be able to describe the end behavior of any polynomial using limit notation.
2. Be able to find the zeros of a polynomial by factoring.
3. Be able to find the zeros of a polynomial using your graphing calculator.
4. Understand how the multiplicity of a zero changes how the graph behaves when it hits the *x*-axis.
5. Use end behavior and multiplicity of zeros to sketch a polynomial by hand.
6. Use the regression capabilities of your calculator to model a cubic and quartic equation.

Our focus today is on polynomials of degree 3 (cubic), degree 4 (quartic), or higher.

When we looked at horizontal asymptotes, we talked about end behavior. End behavior is the behavior of the function (the y – values) as x approaches positive or negative infinity.

If you can remember the graphs of $y = x$, $y = -x$, $y = x^2$, and $y = -x^2$, then you can remember the end behavior of all polynomial functions. End behavior of any *polynomial* function can be described using the "leading coefficient" and highest power.

	Even Power	Odd Power
Positive Coefficient		
Negative Coefficient		

Example 1: Describe the end behavior of the polynomial function using $\lim\limits_{x\to\infty}$ and $\lim\limits_{x\to-\infty}$. Confirm graphically.

a) $f(x) = 3x^6 - 5x + 3$

b) $g(x) = -x^3 + 7x^2 - 8x + 9$

(♬: When the end behavior of a function goes to positive or negative infinity, we will write $\lim\limits_{x\to\infty} f(x) = \infty$ or $-\infty$, but since infinity isn't a real number, we say the limit does not exist)

Zeros of Polynomial Functions

The zeros of a function are the *x*-values that make the function equal 0. These *x*-values are also called the *x*-intercepts. We can use factoring to find these zeros algebraically.

Example 2: Find the zeros of the function algebraically. Support graphically.

a) $f(x) = x^3 - 16x$

c) $h(x) = 4x^3 - 14x^2 + 6x$

Graphical Analysis of Zeros

Example 3: Graph $f(x) = 2x^3 + 3x^2 - 7x - 6$ in a viewing window that shows all of its *x*-intercepts and find all of its zeros.

Definition: Multiplicity of zeros:

The multiplicity of each zero is the number of times the factor occurs in the factored form of the polynomial.

Example 4: List each zero and it's multiplicity. Then graph each on your calculator. What do you notice?

a) $f(x) = x(x-3)^2$

b) $h(x) = (x+1)(x-2)^2(x-4)^3$

At a zero with EVEN multiplicity, the graph of the function will ___________________________________.

At a zero with ODD multiplicity, the graph of the function will ___________________________________.

Putting It All Together

Example 5: State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the *x*-axis at the corresponding *x*-intercept. Using what you know about end behavior and the zeros of the polynomial function, sketch the function.

a) $f(x) = x^2(x+2)^3$

Degree = ______ Sketch:

End Behavior:

Zeros & Multiplicity:

b) $h(x) = -3(x-2)(x+7)^2$

Degree = ______ Sketch:

End Behavior:

Zeros & Multiplicity:

Using Your Calculator's Regression Abilities to Fit Data to a Polynomial

Your calculator can find a polynomial up to degree 4 that best fits the data …

The best fit line (degree 1) = Linear Regression (LinReg)
The best fit quadratic (degree 2) = Quadratic Regression (QuadReg)
The best fit cubic (degree 3) = Cubic Regression (CubicReg)
The best fit 4th degree polynomial = Quartic Regression (QuartReg)

Example 6: Use cubic regression to fit a curve through the four points given in the table.

x	**3**	**4**	**6**	**8**
y	10	15	21	33

2.4 REAL ZEROS OF POLYNOMIAL FUNCTIONS

Learning Targets:

1. Use long division to find factors of a polynomial.
2. Use synthetic division to find linear factors of a polynomial.
3. Apply the remainder theorem to find the function value at a given value of *x*.
4. Completely factor a polynomial using given zeros or a graph of the function.

In the last section, we found the zeros of a polynomial by factoring. In this section we are going to explore different ways to obtain those factors. We will focus on long division and synthetic division.

Example 1: Divide. Write a summary statement in polynomial form.

a) $2x+1\overline{\big)2x^3-3x^2+6x-7}$

b) $\dfrac{2x^4-3x^3+5x-1}{x-2}$

As long as you are dividing by a term that looks like ______________, you can use synthetic division.

Example 2: Divide using synthetic division. Write a summary statement.

a) $\left(x^3+2x^2-5x+6\right)\div\left(x+4\right)$

b) $\left(2x^4-3x^3+5x-1\right)\div\left(x-2\right)$

The Remainder Theorem

If a polynomial is divided by ($x - c$), then the remainder is the same as the function value $f(c)$.

When a polynomial is divided by ($x - c$), we can find the remainder using

a) ____

b) ____

c) ____

The Factor Theorem

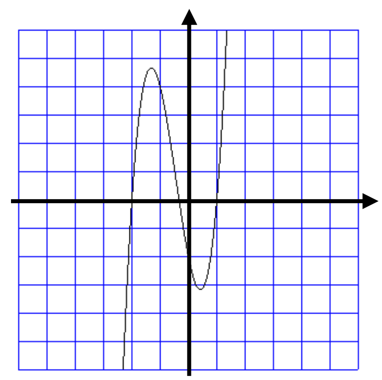
For a polynomial function, $x = c$ is a zero of a function if and only if ($x - c$) is a factor of the function.

A Quick Summary … The following statements are all equivalent:

1. $x = c$ is a solution (or root) of the equation $f(x) = 0$.
2. When $f(x)$ is divided by $(x - c)$ the remainder equals 0.
3. c is a zero of the function $f(x)$.
4. c is an x-intercept of the graph of $f(x)$.
5. $(x - c)$ is a factor of $f(x)$.

Example 3: Determine whether $(x + 2)$ is a factor of $4x^3 + 9x^2 - 3x - 7$.

Example 4: Completely factor $f(x) = 3x^3 + 4x^2 - 5x - 2$.
Use the graph below and synthetic division.



<u>*Rational Zeros Theorem*</u>

Let $f(x)$ be a polynomial function with integral coefficients. The only <u>***possible rational zeros***</u> of $f(x)$ are:

$$\frac{p}{q},$$

where <u>p is a divisor of the constant term</u> and <u>q is a divisor of the leading coefficient</u>.

Example 5: List all possible rational zeros of the function: $f(x) = 6x^3 + 5x^2 - 21x + 10$

a) List all possible rational roots $\dfrac{p}{q}$ … these are the only <u>*possible*</u> rational zeros of the function.

b) Graph the function to see which <u>**are**</u> the zeros of $f(x)$.
… and realize how nice your teacher is for not making you try ALL the numbers in part *c*.

c) Using the graph from part *d* to get you started; find the zeros of $f(x)$ algebraically.
… (use synthetic division and/or factoring)