AP Calculus
3.6 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Find $f'(x)$, if $f(x) = \tan(\cos x)$

2. Find $\dfrac{dy}{dx}$, if $y = \sin\left(\tan\sqrt{\sin x}\right)$

3. Find $\dfrac{ds}{d\theta}$, if $s = 2\theta\sqrt{\sec\theta}$

4. Suppose f and g are differentiable functions with the values given in the table below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	5	e	$\sqrt{2}$
5	2	8	π	7

a) If $h(x) = f(g(x))$, find $h'(2)$.

b) If $h(x) = g(f(x))$, find $h'(2)$.

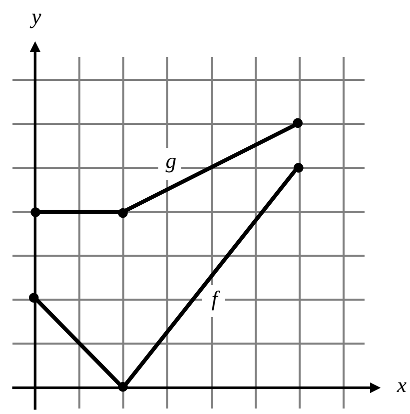
c) If $h(x) = f(f(x))$, find $h'(2)$.

5. Find $\frac{dy}{dt}$, if $y = \tan x$. (No … there are no typos in this problem … what do you notice?)

6. Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$ where f and g are shown in the figure below.

a) Find $r'(1)$.

b) Find $s'(4)$.



7. Find the equation of the tangent line when $x = 4$ on the function $f(x) = \sqrt{25 - x^2}$.

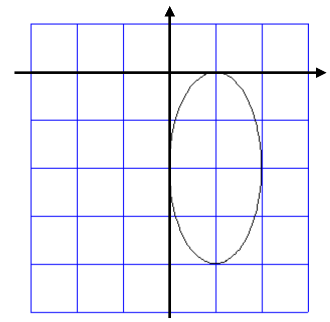
8. Find $\frac{dy}{dx}$ for $y = \sin^4(3x)$.

9. Complete the following problems from the textbook: page 153 - 155 #9, 13 – 31 odd, 53, 55, 63

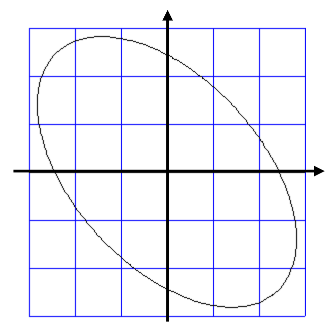
AP Calculus
3.7 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

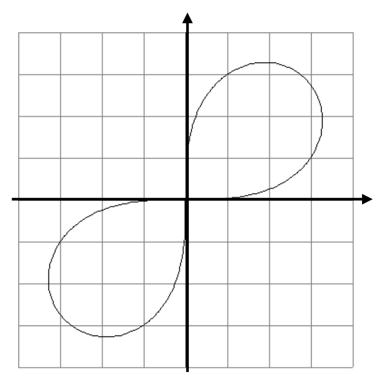
1. Find the points at which the graph of $4x^2 + y^2 - 8x + 4y + 4 = 0$ has a vertical tangent line. (Pretend the picture isn't there until *after* you have found the points!)



2. Find the point(s) (if any) of horizontal tangent lines: $x^2 + xy + y^2 = 6$
(Does your answer make sense given the picture to the right?)



3. Determine the slope of the graph of $3\left(x^2 + y^2\right)^2 = 100xy$ at the point (3, 1).
(Does your answer make sense given the picture to the right?)



Complete the following questions from the textbook: page 162 – 164: #1, 7, 17, 25 – 37 odd, 46, 55, 56

AP Calculus
3.8 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Find the derivative of $h(x) = x\sec^{-1}(3x)$

2. Find the derivative of $f(x) = x\sqrt{1-x^2} + \cos^{-1}(x^3)$

3. Find an equation for the line tangent to the graph of $y = \tan x$ at the point $\left(\frac{\pi}{4}, 1\right)$.

4. Find an equation for the line tangent to the graph of $y = \tan^{-1} x$ at the point $\left(1, \frac{\pi}{4}\right)$.

5. Find $\left(f^{-1}\right)'(2)$ if $f(x) = x^3 + 2x - 1$.

6. Find $\left(f^{-1}\right)'(6)$ if $f(x) = x^3 - \frac{4}{6}$.

7. Complete the following questions from the textbook: p170 #1, 5, 12, 13, 15, 20, 23, 25

AP Calculus
3.9 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Suppose $10 = e^{xy} + x^2 + y^2$, find $\dfrac{dy}{dx}$.

2. Find $g'(t)$ if $g(t) = t^e\left(e^{-t}\right)$

\

3. Find $g'(t)$ if $g(t) = \ln(\ln t)$.

4. Use properties of logarithms to rewrite $h(x)$ and then find $h'(x)$ if $h(x) = \ln\left(\dfrac{1+e^x}{1-e^x}\right)$.

5. Find the first derivative for $y = x^{\ln x}$ (use logarithmic differentiation).

6. Find y' if $y = \frac{x^3}{3^x}$ first using the quotient rule, then using logarithmic differentiation.

7. Solve the following without using a calculator: If $f(x) = \left(x^2 + 1\right)^{(2-3x)}$, then $f'(1) =$

A $-\frac{1}{2}\ln(8e)$ B $-\ln(8e)$ C $-\frac{3}{2}\ln(2)$ D $-\frac{1}{2}$ E $\frac{1}{8}$

8. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

A 0 B $\frac{1}{e}$ C 1 D $\frac{2}{e}$ E $\sec^2(e)$

9. Complete the following questions from the textbook:

page 178 – 179 #1, 4, 5, 7, 8, 11, 12, 13, 16, 17, 21, 22, 26, 29, 30, 33, 37, 40, 41, 45

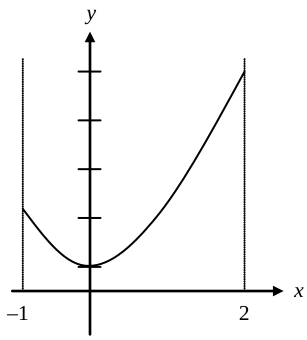
AP Calculus
4.1 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

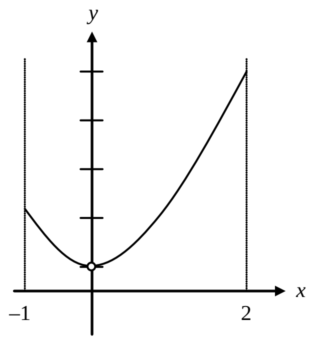
1. State the *hypothesis* of the Extreme Value Theorem.

2. Using the graphs provided, find the minimum and maximum values on the given interval. If there is no maximum or minimum value, explain which part of the hypothesis of the Extreme Value Theorem is not satisfied.

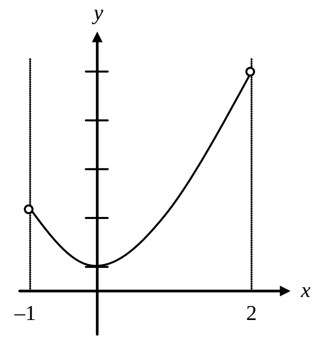
(a) [–1, 2]



(b) [–1, 2]



(c) (–1, 2)



3. When looking for extrema, where do you find the candidates?

Explain why each of the statements in questions 4 – 6 are false.

4. If $f'(5) = 0$, then there is a maximum or a minimum at $x = 5$.

5. If $x = 2$ is a critical number, then $f'(2) = 0$.

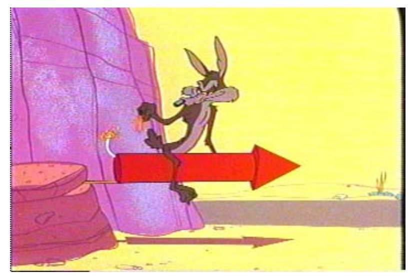
6. An extrema occurs at every critical number.

7. Find the extrema of $h(\theta) = 2\sin\theta - \cos(2\theta)$ for $0 \le \theta \le 2\pi$. Use your graphing calculator to investigate first.

8. Wile E. is after Road Runner again! This time he's got it figured out. Sitting on his ACME rocket he hides behind a hill anxiously awaiting the arrival that "beeping" bird. In his excitement to light the rocket he tips the rocket up. Instead of thrusting himself parallel to the ground where he can catch the Road Runner, he sends himself widely into the air following a path given by function

$$h(t) = .1t^3 - 1.3t^2 + 4.2t + 2,$$

where *h* is the height of the rocket after *t* seconds. The rocket fuel lasts for 10 seconds. At that point, Wile E. Coyote stops suddenly and falls straight down to the ground. What is the highest point reached by Wile E. Coyote?



9. Complete the following problems from the textbook: 4.1 pages 193 – 195 #5 – 10, 12, 13, 16, 17, 21, 23, 26, 44, 48
4. 4 page 226 #2, 6, 9

AP Calculus
4.2 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. State the MVT 2 ways …

   a) … in words

   b) … algebraically

2. Let $f(x) = -2x^2 + 14x - 12$ on the interval [1, 6]

   a) How do you know this function satisfies the hypothesis of the MVT?

   b) Find the value of *c* guaranteed by the MVT.

The last example is a special version of the Mean Value Theorem called Rolle's Theorem. In fact, the proof of the Mean Value Theorem can be done quite easily, if you prove Rolle's Theorem first. Rolle's Theorem basically states that if the function is continuous on the closed interval and differentiable on the open interval AND the values of the function at the endpoints are equal, then there must exist at least one point in the interval where the derivative is zero.

3. Summarize how we will use calculus to determine whether a function is increasing or decreasing.

4. Make a sign chart for the following functions:

   a) $f(x) = (x-3)^2(x+4)(x+7)$

   b) $g(x) = \dfrac{5(2x-7)}{(x+1)(3x-5)}$

5. Find the critical numbers of f and the intervals where f is increasing or decreasing if $f(x) = x^3 - 6x^2 + 15$.

6. The Profit P in dollars made by a fast food restaurant selling x hamburgers is given by

$$P = 2.44x - \frac{x^2}{20000} - 5000, \quad 0 \le x \le 35000.$$

a) Find the open intervals on which P is increasing or decreasing

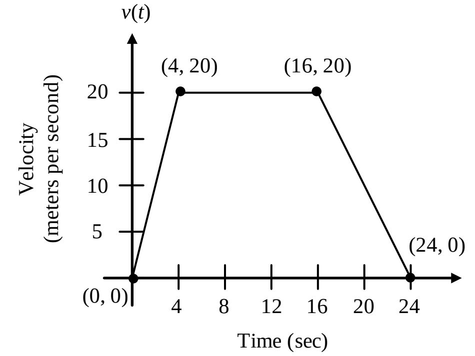
b) Find the maximum profit.

7. If you know that the acceleration of gravity is $-32\, {}^{ft}/_{s^2}$, for an falling object, we could write the acceleration of the object at time t as $a(t) = -32$.

a) Find a function for the velocity of the object at time t. What does the constant equal (in words)?

b) Find a function for the position of the object at time t. What does the constant equal (in words)?

8. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph below.



a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain its meaning.

[Obviously we haven't used this symbol yet, nor have we talked about how to get it … so here's a couple of hints …]

i) If I told you the notation $\int_0^{24} v(t)\,dt$ *only asked you to find the antiderivative of the velocity function, you should be able to* *<u>use correct units</u>.*

ii) If I told you that all the notation $\int_0^{24} v(t)\,dt$ *means for this problem is to find the area under the given curve, you should then be able to answer the question AND* *<u>explain the meaning</u>* *of* $\int_0^{24} v(t)\,dt$.

b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

\

9. Complete the following questions from the textbook:

4.2: page 202 – 203 #1, 3, 6, 10, 11, 15, 17, 19, 21, 25, 29, 31, 33, 38, 43

4.4: page 226 – 228 #10, 20, 23

AP Calculus
4.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Circle the correct word that completes the following statements:
   a) When the graph of f is increasing, f' is (positive , negative).

   b) When the graph of f is decreasing, f' is (positive , negative).

   c) The graph of f is concave upward when f'' is (positive , negative).

   d) The graph of f is concave downward when f'' is (positive , negative).

   e) The graph of f is concave upward when f' is (increasing , decreasing).

   f) The graph of f is concave downward when f' is (increasing , decreasing).

2. Use the function $y = 3x - x^3 + 5$. [No calculator allowed]

   a) Where is the function increasing? Justify your response.

   b) Where is the function decreasing? Justify your response.

   c) Where is the function concave up? Justify your response.

   d) Where is the function concave down? Justify your response.

   e) Where are the point(s) of inflection? Justify your response.

   f) Find ALL extrema and justify your response.

   g) Create a *sketch* of the function.

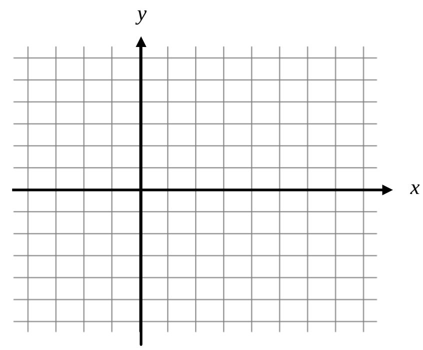
3. [2005 AP Calculus AB Free Response #4 … No Calculator Allowed]

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval [0, 4]. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

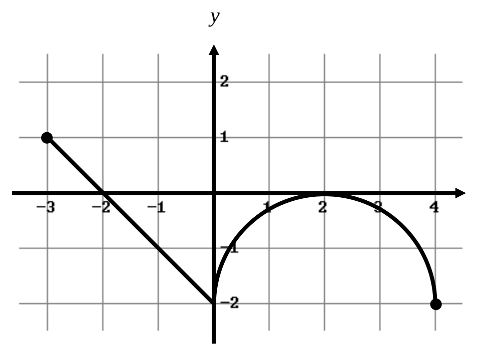
a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

b) On the axes provided, sketch the graph of a function that has all the characteristics of f.



4. [2003 AP Calculus AB Free Response #4 … No Calculator Allowed] Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.

a) On what intervals, if any, is f increasing? Justify your answer.



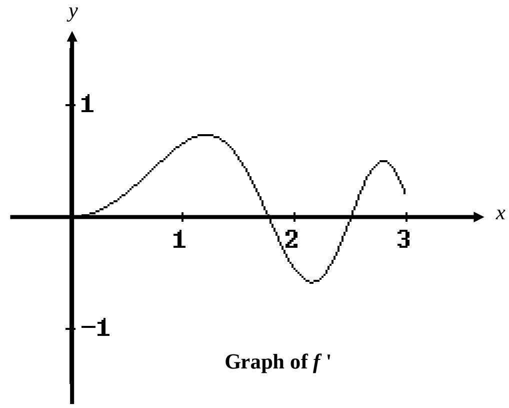
Graph of f'

b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

c) Find an equation for the line tangent to the graph of f at the point (0, 3).

5. [2006 AP Calculus AB Form B Free Response #2 … Calculator Required] Let f be a function defined for $x \ge 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown below.

a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.



Graph of f'

b) Write an equation for the line tangent to the graph of f at $x = 2$.

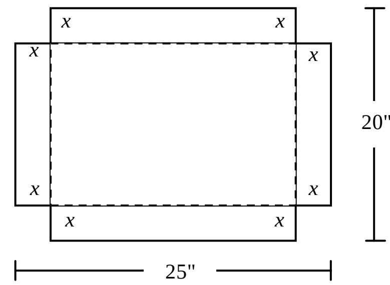
6. [2004 AP Calculus AB Free Response Question #4 … No Calculator Allowed]
Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

a) Show $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$

b) Show that there is a point P with x – coordinate 3 at which the line tangent to the curve P is horizontal. Find the y – coordinate of P.

c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part b. Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

7. An open-top box is to be made by cutting congruent squares of side length x from the corners of a 20- by 25-inch sheet of tin and bending up the sides (see figure below). How large should the squares be to make the box hold as much as possible? What is the resulting volume?



8. If g is a differentiable function such that $g(x) < 0$ for all real numbers x, and if $f'(x) = (x^2 - 9)g(x)$, which of the following is true?

A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 3$.

B) f has a relative minimum at $x = -3$ and a relative maximum at $x = 3$.

C) f has relative minima at $x = -3$ and at $x = 3$.

D) f has relative maxima at $x = -3$ and at $x = 3$.

E) It cannot be determined if f has any relative extrema.

9. Complete the following questions from the textbook:
4.3: p215 – 217 #1, 4, 7, 9, 13, 20, 27, 35, 39, 49, 51,
4.4 p228 – 231 #30, 40, 56

You should also start reviewing for your chapter 3.6 – 4.4 exam by completing the following questions from the textbook:
Page 181 #5 – 11, 32 – 48 even, 51, 55, 62, 67, 68, 70
Page 256 #1, 3, 5, 17, 21, 23, 25, 26, 27, 30, 31, 32, 33, 35, 36, 37, 40, 45, 49, 52