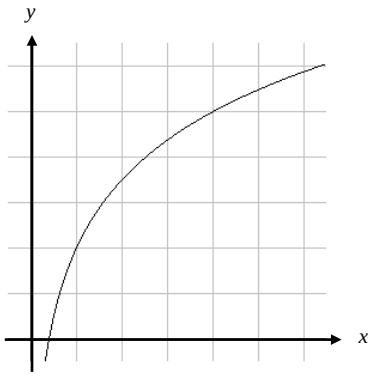
AP Calculus
3.1 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. What is the definition of a derivative?

2. What is the alternative definition of a derivative?

3. Identify or sketch each of the quantities on the figure to the right.



   a) $f(1)$ and $f(4)$

   b) $f(4)-f(1)$

   c) $y=\frac{f(4)-f(1)}{4-1}(x-1)+f(1)$

   d) Insert the proper inequality symbol (< or >) between the given quantities.

      i) $\frac{f(4)-f(1)}{4-1} \square \frac{f(4)-f(3)}{4-3}$

      ii) $\frac{f(4)-f(1)}{4-1} \square f'(1)$

4. The figure to the right shows the graph of g'.

   a) $g'(0)=$

   b) $g'(3)=$

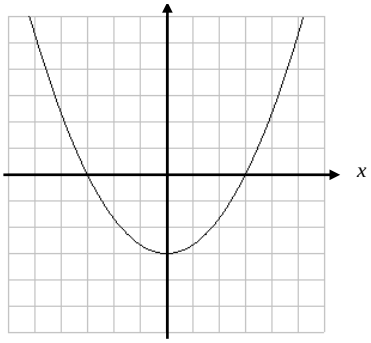
   c) What can you conclude about the graph of g knowing that $g'(1)=-\frac{8}{3}$?

   d) What can you conclude about the graph of g knowing that $g'(4)=\frac{7}{3}$?

   e) Is $g(6)-g(4)$ positive or negative? Explain.

   f) Is it possible to find g (2) from the graph? Explain.

The graph of g'



5. Assume that $f'(c)=3$. Find $f'(-c)$ given the following conditions:

   a) f is an odd function.

   b) f is an even function.

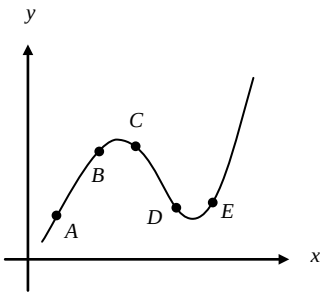
6. Use the graph of f at the right to answer each question.

a) Between which two consecutive points is the average rate of change of the function greatest?

b) Is the average rate of change of the function between A and B greater than or less than the instantaneous rate of change of B?

c) Sketch a tangent line to the graph between the points B and C such that the slope of the tangent line is the same as the average rate of change of the function between B and C.

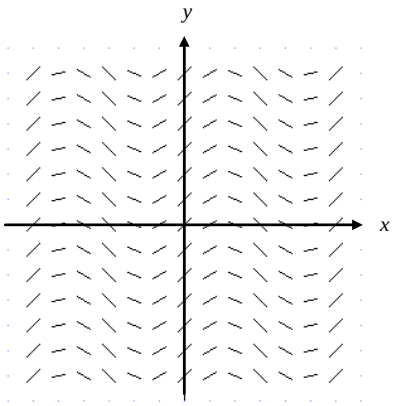
d) Give any sets of consecutive points for which the average rates of change of the function are approximately equal.



7. Sketch a function whose derivative is ALWAYS negative.

8. Sketch a function whose derivative is ALWAYS positive.

9. Use the slope field below to sketch at least two possible graphs of the function f.



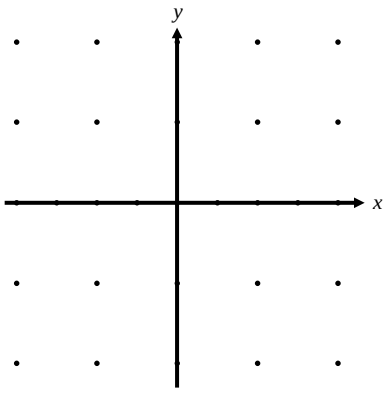
The equation for this slope field is $\frac{dy}{dx} = \cos x$.

What do you think the original function was?

10. Use the grid to the right.

a) Draw the slope field for $\frac{dy}{dx} = -\frac{x}{y}$.

b) If the point (0, –1) is on the graph of y, draw the graph of y.



12. Complete the following questions from the textbook: pages 105 – 108: #1 – 8, 13 – 17, 21, 26, 32, 44

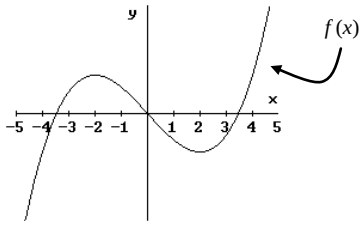
AP Calculus
3.2 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. If $f(x) = 2 + |x+3|$ for all values of x, then the value of the derivative $f'(x)$ at $x = 3$ is

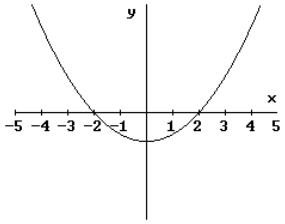
A) –1 B) 0 C) 1 D) 2 E) nonexistent

2. The graph of $f(x)$ is shown in the figure below.

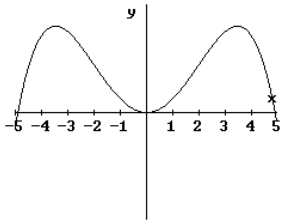


Which of the following could be the graph of $f'(x)$?

A



B



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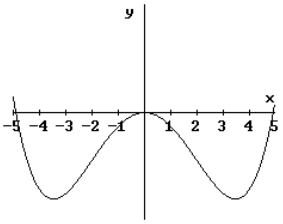
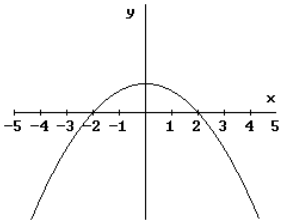
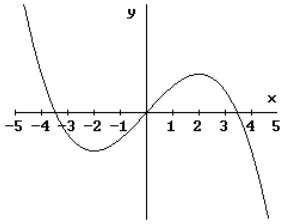


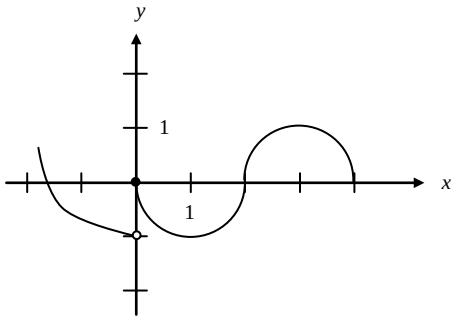
D



E



3. The graph of the function f shown in the figure below has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, –1) and (3, 1).



For what values of x, $-2 < x < 4$, is f not differentiable?

A) 0 only B) 0 and 2 only C) 1 and 3 only D) 0, 1, and 3 only E) 0, 1, 2, and 3

4. If f is a function such that $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=0$, which of the following must be true?

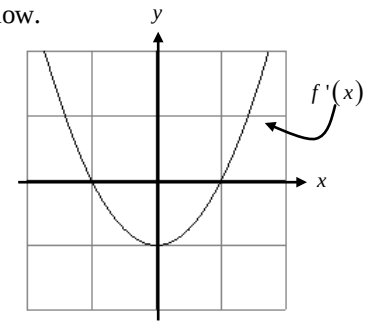
A) The limit of $f(x)$ as x approaches 2 does not exist.
B) f is not defined at $x = 2$.
C) The derivative of f at $x = 2$ is 0.
D) f is continuous at $x = 0$.
E) $f(2) = 0$

5. Let f be a function such that $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=5$. Which of the following must be true?

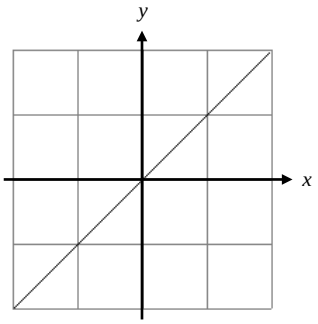
I. f is continuous at $x = 2$.
II. f is differentiable at $x = 2$.
III. The derivative of f is continuous at $x = 2$.

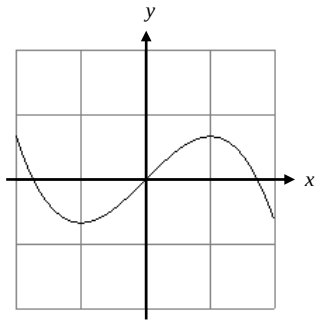
A) I only B) II only C) I and II only D) I and III only E) II and III only

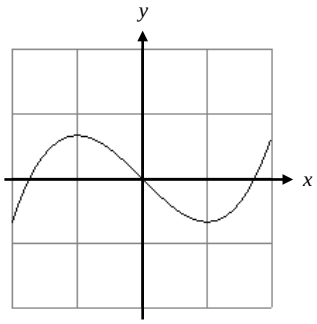
6. The graph of the ***derivative*** of f is shown below.

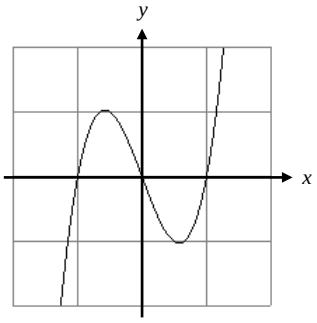


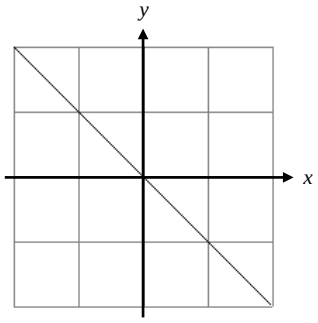
Which of the following could be the graph of f?

A)


B)


C)


D)


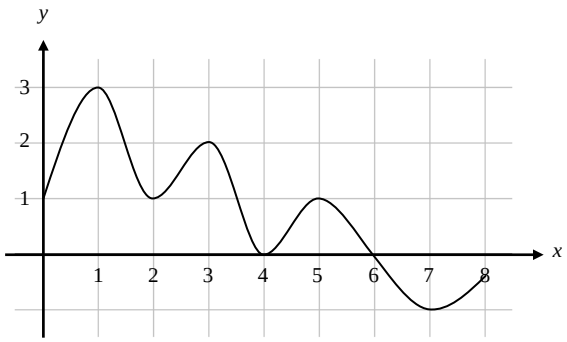
E)


7. Let f be a function that is differentiable on the open interval (0, 10). If $f(2) = -5$, and $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent line.
III. For some c, $2 < c < 5$, $f(c) = 3$.

A) none B) I and II only C) I only D) I and III only E) I, II, and III

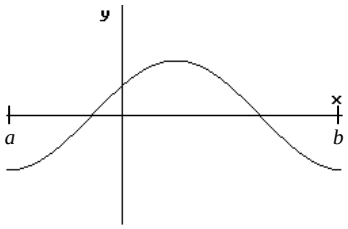
8. The function f is defined on the closed interval [0, 8]. The graph of its derivative f' is shown below.



The point (3, 5) is on the graph of $f(x)$. An equation of the tangent line to the graph of f at (3, 5) is

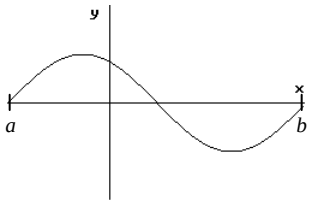
A) $y = 2$ B) $y = 5$ C) $y - 5 = 2(x - 3)$ D) $y + 5 = 2(x - 3)$ E) $y + 5 = 2(x + 3)$

9. The graph of f is shown below.

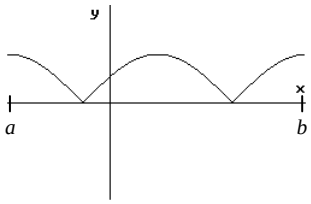


Which of the following could be the graph of the derivative of f?

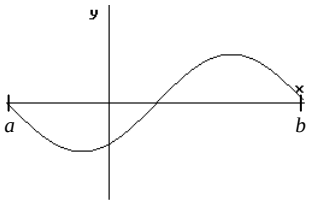
A.



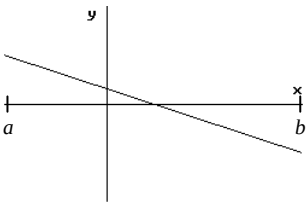
B.



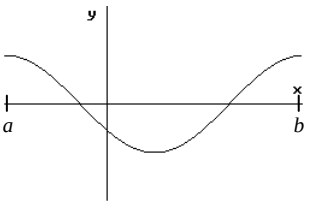
C.



D.



E.



10. Complete the following questions from the textbook: pages 114 – 115: #5 – 10, 11 – 16 (calculator), 39

11. Complete the Worksheet: Graphs of f, f', f'', and F

AP Calculus
3.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Solve for *a* and *b* in order for $f(x)$ to be both continuous and differentiable at $x = 1$.

$$f(x)=\begin{cases} x^2+2 & ;x\le 1 \\ a\left(x-\frac{1}{x}\right)+b & ;x>1 \end{cases}$$

2. Solve for *a* and *b* in order for $g(x)$ to be both continuous and differentiable at $x = 0$.

$$g(x)=\begin{cases} ax+b & ;x>0 \\ 1-x+x^2 & ;x\le 0 \end{cases}$$

3. For *a* – *d*, find $f'(2)$ given the following information:

$$g(2)=3 \quad g'(2)=-2$$
$$h(2)=-1 \quad h'(2)=4$$

a) $f(x)=2g(x)+h(x)$

b) $f(x)=4-h(x)$

c) $f(x)=g(x)h(x)$

d) $f(x)=\dfrac{g(x)}{h(x)}$

4. Find all points where the graph of $y = x^4 - 5x^3 - 3x^2 + 13x + 10$ has a horizontal tangent line.

5. Find the equation of the tangent line to the graph of $f(x) = (x^3 - 3x + 1)(x + 2)$ at the point (1, –3).

6. Let $f(x) = (3x^3 + 4x^2)(2x^4 - 5x)$. Find $f'(x)$ without using the product rule first, then using the product rule.

7. An equation of the line tangent to the graph of $y = \dfrac{2x+3}{3x-2}$ at the point (1, 5) is

A) $13x - y = 8$ B) $13x + y = 18$ C) $x - 13y = 64$ D) $x + 13y = 66$ E) $-2x + 3y = 13$

8. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

A) 3 B) 4 C) 6 D) 8 E) 12

9. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

A) $\frac{1}{4}$ B) 1 C) 4 D) $\frac{1}{\sqrt{2}}$ E) $\frac{1}{2\sqrt{2}}$

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

A) –2 B) $\frac{1}{6}$ C) $\frac{1}{2}$ D) 2 E) 6

11. Which of the following is an equation of the tangent line to $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

A) $y = 8x - 5$ B) $y = x + 7$ C) $y = x + .763$ D) $y = x - .122$ E) $y = x - 2.146$

12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ C) $\left(1, -\frac{1}{4}\right)$ D) $\left(1, \frac{1}{2}\right)$ E) $(2, 2)$

13. If u, v, and w are nonzero differentiable functions, then the derivative of $\dfrac{uv}{w}$ is

A) $\dfrac{uv' + u'v}{w'}$

B) $\dfrac{u'v'w - uvw'}{w^2}$

C) $\dfrac{uvw' - uv'w - u'vw}{w^2}$

D) $\dfrac{u'vw + uv'w + uvw'}{w^2}$

E) $\dfrac{uv'w + u'vw - uvw'}{w^2}$

14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

A) 0.4 B) 0.5 C) 2.6 D) 3.4 E) 5.5

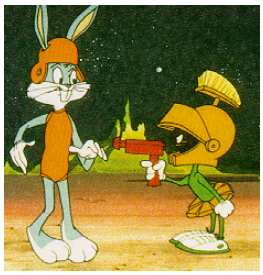
15. Complete the following questions from the textbook:
pages 124 – 126 #1 – 9 (odd), 14, 17, 18, 20, 21, 23, 26, 27, 30, 37 – 40, 42, 47, 56

AP Calculus
3.4 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Boyle's Law states that if the temperature of a gas remains constant, its pressure is inversely proportional to its volume. Show that the rate of change of the pressure is inversely proportional to the square of the volume.

2. Once again trying to blow up earth because it interferes with his view of Venus, Marvin the Martian lands on the moon. Bugs Bunny, as always, interferes with his plan. Chasing Bugs, Marvin fires a warning shot straight up into the air with his Acme Disintegration Pistol. The height (in feet) after t seconds of the shot is given by



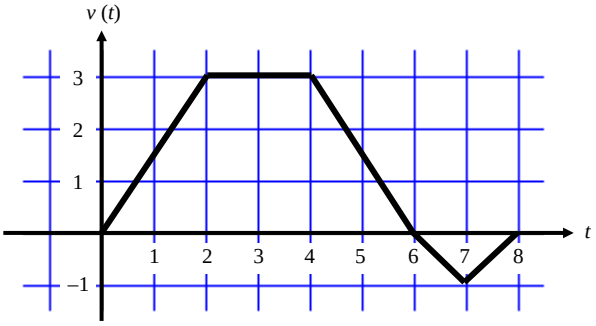
$$s(t) = -2.66t^2 + 135t + 3.$$

a) Find the velocity and acceleration as functions of time.
(What is the meaning of the acceleration function?)

b) How long will it take for Marvin's shot to reach its maximum height?

c) What is the maximum height for Marvin's shot?

3. A bug begins to crawl up a vertical wire at time $t = 0$. The velocity, v, of the bug at time t, $0 \le t \le 8$ is given by the function whose graph is shown below.



At what value of t does the bug change direction?

A) 2 B) 4 C) 6 D) 7 E) 8

4. If the position of a particle on the x – axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

A) –45 B) –30 C) –15 D) –10 E) –5

5. A particle moves along the x – axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

A) 1 B) 2 C) 3 D) 4 E) 5

6. Rocket *A* has a positive velocity *v* (*t*) after being launched upward from an initial height of 0 feet at time *t* = 0 seconds. The velocity of the rocket is recorded for selected values of *t* over the interval $0 \le t \le 80$ seconds as shown in the table below.

T (sec)	0	10	20	30	40	50	60	70	80
v (*t*) (ft/sec)	5	14	22	29	35	40	44	47	49

a) Find the average acceleration of Rocket *A* over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

b) Using the data, find an estimate for $v'(35)$. Indicate units of measure.

7. A particle moves along the *x* – axis so that its position at any time $t \ge 0$ is given by the function $x(t) = t^3 - 12t + 1$, where *x* is measured in feet and *t* is measured in seconds.

a) Find the displacement during the first 3 seconds.

b) Find the average velocity during the first 3 seconds.

c) Find the instantaneous velocity when *t* = 3 seconds.

d) Find the acceleration of the particle when *t* = 3 seconds.

e) When is the particle moving left?

f) At what value or values of *t* does the particle change directions?

g) When is the particle speeding up?

8. The cost involved in maintaining annual inventory for a certain manufacturer is given by $C(x) = \frac{1,008,000}{x} + 6.3x$.
Where *x* is the number of items stored. Find the marginal cost of storing the 351^{st} item.

9. Complete the following questions from the textbook: pages 135 – 140 #2 – 5, 8 – 13, 18, 19, 23, 25, 27, 37

AP Calculus
3.5 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. A spring is bobbing up and down so that its position at any time $t \geq 0$ is given by $s(t) = -4\sin t$.

   a) What is the initial position of the spring?

   b) Which way is the particle moving to start? Justify your response.

   c) At $t = \frac{5\pi}{4}$, is the spring moving up or down? Justify your response.

   d) Is the spring speeding up or slowing down at $t = \frac{5\pi}{4}$? Justify your response.

2. If $y = \sec x$, find $\frac{d^2y}{dx^2}$.

3. If $f(x) = \sin x$, find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$. What do you think the function $f^{(100)}(x)$ is?

4. $\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} =$

A) 0 B) 1 C) $\sin x$ D) $\cos x$ E) nonexistent

5. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is

A) $y = 2x + 1$ B) $y = x + 1$ C) $y = x$ D) $y = x - 1$ E) $y = 0$

6. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

A) $\sec x \csc x$ B) $\sec x - \csc x$ C) $\sec x + \csc x$ D) $\sec^2 x - \csc^2 x$ E) $\sec^2 x - \csc^2 x$

7. If $f(x)=\dfrac{x}{\tan x}$, then $f'\left(\frac{\pi}{4}\right)=$

A) 2 B) $\frac{1}{2}$ C) $1+\frac{\pi}{2}$ D) $\frac{\pi}{2}-1$ E) $1-\frac{\pi}{2}$

8. [Calculator] A particle moves along a line so that at time t, $0 \le t \le \pi$, its position is given by $s(t)=-4\cos t-\dfrac{t^2}{2}+10$. What is the velocity of the particle when its acceleration is zero?

A) –5.19 B) 0.74 C) 1.32 D) 2.55 E) 8.13

9. If $f(x)=\sin x$, then $f'\left(\frac{\pi}{3}\right)=$

A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\sqrt{3}$

10. [Calculator] A body is moving in simple harmonic motion (up/down) with position $s(t)=3+\cos t$, where $0 \le t < 2\pi$.

a) Find the velocity, $v(t)$, of the object at any time t.

b) Find the zeros of $v(t)$.

c) Find the acceleration, $a(t)$, of the object at any time t.

d) Find the zeros of $a(t)$.

e) When is the object stopped? Justify your response.

f) When does the object change direction? Justify your response.

g) When does the object speed up? Justify your response.

11. Complete the following questions from the textbook: pages 146 – 147 #1, 4, 5, 9, 16, 22, 23, 27, 30, 32, 36, 37, 40, 41

… you should also begin reviewing for your chapter test 3.1 – 3.5: Review: #1 – 4, 43, 53, 57, 59 – 63, 71, 73
This isn't due until the day of your exam.