Calculus AB
Chapter 3.6 – 4.4 Review

CONCEPTS:

1. When looking for absolute extrema, where do the possible extrema exist, and how do you find them?

2. How do you justify relative extrema?

3. How do you justify that a function is increasing or decreasing?

4. How do you justify that a function is concave up or concave down?

5. How do you justify that a function has a point of inflection?

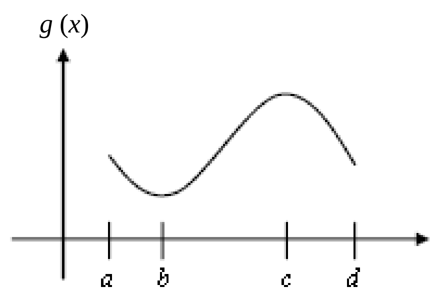
6. Using the graph of $g(x)$ below, determine the signs of $g'(x)$ and $g''(x)$ at each point. Explain your reasoning.

At $x = a$ …

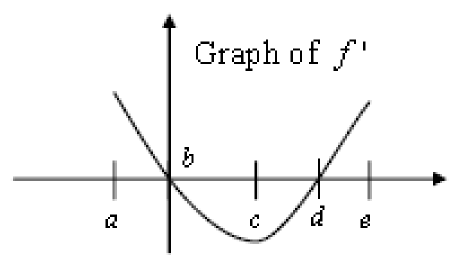
At $x = b$ …

At $x = c$ …

At $x = d$ …



7. Given the graph of f' below answer each of the following questions, and justify your response with a statement that contains the phrase "since f' ____________________ …"



a) When is f increasing?

b) When is f decreasing?

c) When is f concave up?

d) When is f concave down?

e) When does f have a relative maximum?

f) When does f have a relative minimum?

g) When does f have a point of inflection?

SKILLS:

8. [Calculator Allowed] If $f(x)$ has an inverse, then $f\left(f^{-1}(x)\right)=x$. Find $\left(f^{-1}\right)'(2)$ if $f(x)=x^3+2x-1$.

9. Find the value of *c* guaranteed by the MVT for $f(x)=4x^2+5x$ on the interval [–2, 1].

10. [Calculator Allowed] Find the value of *c* guaranteed by the MVT for $f(x)=\sin x$ on the interval [4, 5].
[♬: For those of you doing this problem algebraically, the answer is NOT $c\approx 1.774\ldots$ Why?]

11. Find the following derivatives:

a) $y=\sin^{-1}\left(3x^2\right)$

b) $y=\tan^{-1}\left(\sin x\right)$

c) $y=\sec^{-1}\left(\frac{1}{x}\right)$

d) $y=5^{x^{-3}-8}$

e) $y=e^{8x}$

f) $y=\log_4\left(\sqrt{9x^3-2}\right)$

g) $y=\ln\left(7x^2+3\right)$

h) $y=3^{\sec(x)}$

i) $y=e^{\ln x}$

While none of the previous derivative questions included product and quotient rules, you should be able to combine these rules with any rules we have learned before. See your quiz from 3.8 and 3.9 for examples.

12. Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
–1	0	–1	2	1
0	–1	–3	–2	4

Find the first derivative of the following combinations at the given value of x.

a) $f(g(x))$ at $x = -1$

b) $f^2(x)g^3(x)$ at $x = 0$

c) $g(f(x))$ at $x = -1$

d) $g(x + f(x))$ at $x = 0$

13. Find $\frac{dy}{dx}$ if $x^2y + 3y^2 = x - 2$

14. Find $y'''(x)$ if $y = (4x+1)^{10}$

15. Suppose $y = x^3 - 3x$. [No Calculator]

a) Find the zeros of the function.

b) Determine where y is increasing or decreasing and justify your response.

c) Determine all local extrema and justify your response.

d) Determine the points where y is concave up or concave down, and find any points of inflection. Justify your responses.

e) Use all your information to sketch a graph of this function.

16. If $f'(x) = x^2 - 9x + 1$, what does $f(x)$ equal?

17. Suppose the acceleration of an object in terms of time is given by $a(t) = 5$.

a) What is the velocity function if $v(2) = 10$?

b) Using your velocity function from part *a*, what is the position function if $s(0) = 5$?

18. Suppose $\frac{d^2y}{dx^2} = x^3 - 4x^2$. Justify each response below.

a) Where is y concave up?

b) Where is y concave down?

c) Are there any inflection points on y? If so, where?

SKILLS AND CONCEPTS APPLIED

19. [Calculator Allowed] The derivative of $h(x)$ is given by $h'(x) = 2\cos\left(x - \frac{\pi}{6}\right) + 1$ on the interval $[-2\pi, 2\pi]$. Justify EVERY response.

a) Where is $h(x)$ increasing?

b) Where is $h(x)$ concave down?

c) Find all extrema of $h(x)$ on the interval $[-2\pi, 2\pi]$.

d) Does $h(x)$ have a point(s) of inflection? If so, where?

20. Find the maximum area of a rectangle inscribed under the curve $f(x) = \sqrt{16 - x^2}$.

21. [Calculator Allowed] A rectangle is inscribed under one arch of $y = 8\cos(0.3x)$ with its base on the *x*-axis and its upper two vertices on the curve symmetric about the *y*-axis. What is the largest area the rectangle can have?

22. The function f is continuous on [0, 3] and satisfies the following:

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3
f	0	Neg	–2	Neg	0	Pos	3
f'	–3	Neg	0	Pos	DNE	Pos	4
f''	0	Pos	1	Pos	DNE	Pos	0

a) Find the absolute extrema of f and where they occur.

b) Find any points of inflection.

c) Sketch a possible graph of f.

Go back and Review the questions from your assignments in this chapter … especially those in section 4.3.

Understand your notecards!