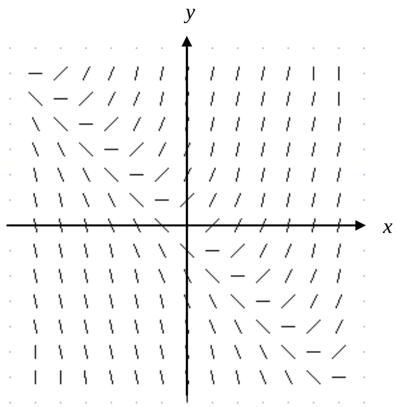
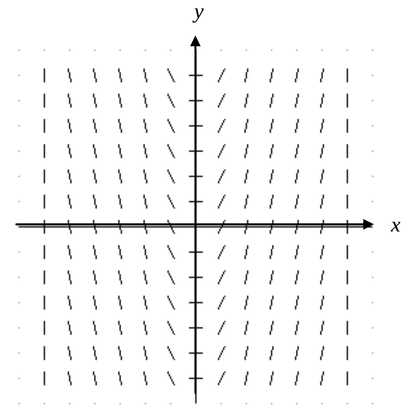
AP Calculus
6.1 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

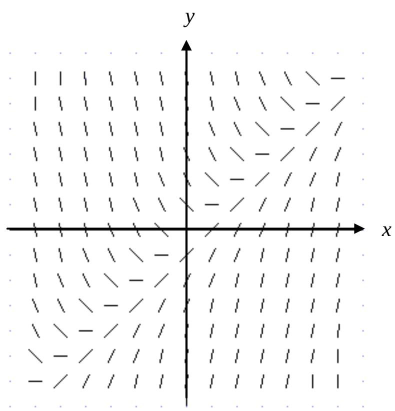
1. Match the six slope fields shown below to their differential equations. Explain each choice.

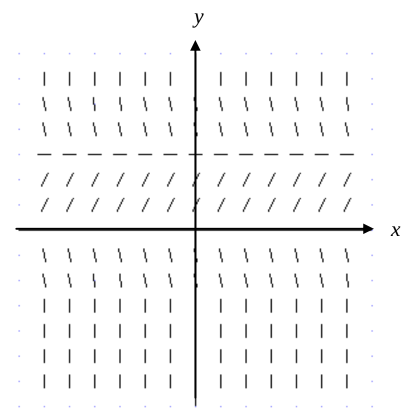




$$\frac{dy}{dx} = x - y$$

$$\frac{dy}{dx} = 2x$$

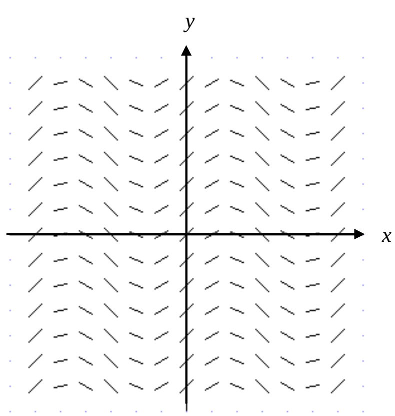


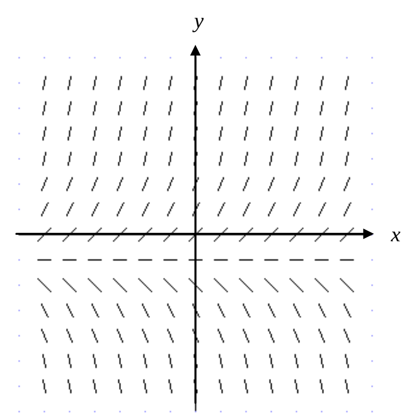


$$\frac{dy}{dx} = 1 + y$$

$$\frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx} = x + y$$

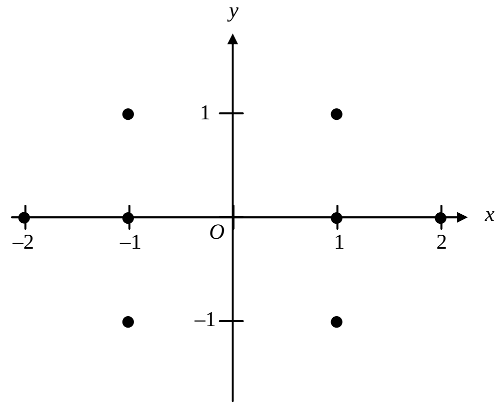




$$\frac{dy}{dx} = y\left(3 - y\right)$$

2. [No Calculator] Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



If the solution to a differential equation is continuous then an initial condition pins down the solution on the entire domain (restricted by the differential equation and the general solution). However, if the solution is NOT continuous, then the initial condition only pins down the continuous piece of the solution curve that passes through the given point.

b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

3. Let f be a function such that $f''(x) = 6x + 12$.

a) Find $f(x)$ if the graph of f is tangent to the line $4x - y = 5$ at the point $(0, -5)$.

b) Find the average value of $f(x)$ on the closed interval $[-1, 1]$.

4. Solve the initial value problem $\frac{d^2y}{dx^2} = 2 - 6x$ given that $y(0) = 1$ and $y'(0) = 4$.

5. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) = ?$ ***Justify your response***. (calculator ok)

A) 0.048

B) 0.144

C) 5.827

D) 23.308

E) 1,640.250

6. The graph of f is shown in the figure below. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) = ?$
Justify your response without using any calculator.

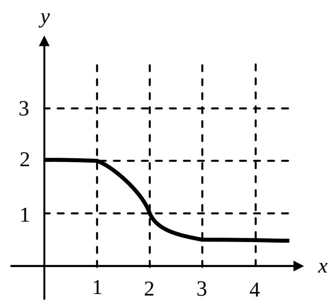
A) 0.3

B) 1.3

C) 3.3

D) 4.3

E) 5.3



6. Let $f(x) = \int_0^{x^2} \sin t\, dt$. At how many points in the interval $\left[0, \sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval? ***Justify your response***. (calculator ok)

A) Zero

B) One

C) Two

D) Three

E) Four

7. Complete the following questions from the textbook: Page 327: #1, 3, 5, 6, 12, 15, 18, 31, 34
(Want more practice? … try the following from the textbook: page 327: #2, 4, 11, 13, 14, 16, 17, 19, 20)

AP Calculus
6.2 Worksheet

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Indefinite Integrals (Straight substitution)

1. $\int \frac{2x}{\sqrt{x^2+6}}\,dx$

2. $\int \frac{e^x}{e^x+4}\,dx$

Definite Integrals (straight substitution)

3. $\int_1^{\sqrt{2}} x \cdot 2^{-x^2}\,dx$

4. $\int_e^{e^2} \frac{1}{x\ln x}\,dx$

5. True or False: $\int_0^{\pi/4} \tan^3(x)\sec^2(x)\,dx = \int_0^{\pi/4} u^3\,du$

Algebraic Techniques

6. $\int \frac{e^x+4}{e^x}\,dx$

7. $\int_0^1 \frac{3\,dx}{(x+1)\sqrt{x^2+2x}}$

8. $\int \frac{dx}{x^2 - 4x + 4}$

9. $\int \frac{x + 2\sqrt{x-1}}{2x\sqrt{x-1}}\, dx$

10. $\int \frac{dx}{\sqrt{-x^2 + 4x - 3}}$

11. $\int \frac{x^5 - 35x}{x^2 + 6}\, dx$

Inv Trig Function examples

12. $\int \frac{dx}{2 + 9x^2}$

13. $\int \frac{dx}{\sqrt{e^{2x} - 1}}$

More Practice … Use another sheet of paper to complete …

14. $\int \frac{e^x}{1+2e^x}\,dx$

15. $\int \sec^2(2x)\,dx$

16. $\int \sec^2(3x)e^{\tan(3x)}\,dx$

17. $\int \frac{x}{2x^2+1}\,dx$

18. $\int e^x\left(2+e^x\right)^{1/2}\,dx$

19. $\int x^2\cos\left(x^3\right)dx$

20. $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

21. $\int \frac{\tan^{-1}x}{1+x^2}\,dx$

22. $\int \csc^2(3x+5)\,dx$

23. $\int \frac{x+1}{\left(x^2+2x+7\right)^3}\,dx$

24. $\int \frac{x}{x^2-4}\,dx$

25. $\int x\tan^2\left(x^2\right)dx$

26. $\int \cos(3x)e^{\sin(3x)}\,dx$

27. $\int \frac{1}{x\ln(3x)}\,dx$

28. $\int \frac{\sin(3x)}{1+\cos(3x)}\,dx$

29. $\int \frac{1}{x^2-2x+17}\,dx$

30. $\int \frac{1}{\sqrt{1-9x^2}}\,dx$

31. $\int x\csc\left(3x^2\right)\cot\left(3x^2\right)dx$

32. $\int \frac{1-e^{-x}}{x+e^{-x}}\,dx$

33. $\int \frac{x^2-1}{x^2+1}\,dx$

34. $\int (x+1)\sqrt{2-x}\,dx$

35. $\int \frac{x+2}{\sqrt{4-x^2}}\,dx$

36. $\int_0^2 \sqrt{4x+1}\,dx$

37. $\int_{-1}^1 \frac{1}{1+x^2}\,dx$

38. $\int_0^1 \frac{1}{(2x+3)^3}\,dx$

39. $\int_2^{e+1} \frac{x}{(x-1)^2}\,dx$

40. $\int_0^\pi \sin\left(\frac{x}{2}\right)dx$

41. $\int_{-\pi}^{\pi} x\sin\left(x^2\right)dx$

42. $\int_{-1}^0 \frac{2}{6x-1}\,dx$

43. $\int_1^5 \frac{(\ln x)^{1/2}}{x}\,dx$

44. $\int_0^1 xe^{-x^2}\,dx$

45. $\int_0^2 \left(2^x+x^2\right)dx$

(NOT Required) Want More Practice? … Try these from the textbook …Page 337: 1 – 6 all, 17 – 42 , 53 – 66

AP Calculus
6.4 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

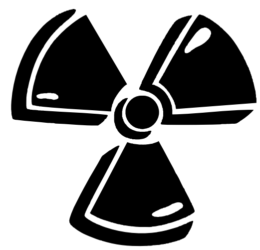
1. Suppose the rate of change of ☺ is proportional to the amount of ☺ present.

a) Write the differential equation that this statement represents.

b) Solve the differential equation from part *a* … do not skip ANY steps.

Remember … if you MUST know how to solve differential equations like the one above, but you may jump straight to the solution if you are solving problems.

2. **Radioactive Decay**: The rate at which a radioactive element decays (as measured by the number of nuclei that change per unit of time) is approximately proportional to the amount of nuclei present. Suppose that 10 grams of the plutonium isotope Pu-239 was released in the Chernobyl nuclear accident. How long will it take for the 10 grams to decay to 1 gram? [Pu-239 has a half life of 24,360 years]



3. Bacteria in a certain culture increase at rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

A) $\dfrac{3\ln 3}{\ln 2}$

B) $\dfrac{2\ln 3}{\ln 2}$

C) $\dfrac{\ln 3}{\ln 2}$

D) $\ln\left(\dfrac{27}{2}\right)$

E) $\ln\left(\dfrac{9}{2}\right)$

4. If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

A) $-\frac{1}{2}\ln 2$

B) $-\frac{1}{4}$

C) $\frac{1}{2}\ln 2$

D) $\frac{\sqrt{2}}{2}$

E) $\ln 2$

5. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

A) 4.2 pounds

B) 4.6 pounds

C) 4.8 pounds

D) 5.6 pounds

E) 6.5 pounds

6. During a certain epidemic, the number of people that are infected at any time increases at rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

A) 343

B) 1,343

C) 1,367

D) 1,400

E) 2,057

7. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

A) 0.069

B) 0.200

C) 0.301

D) 3.322

E) 5.000

8. Complete the following questions from the textbook: page 357: #1, 3, 5, 6, 7, 8, 9, 10, 42, 43