AP Calculus
4.5 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Consider the function $y = \sin x$.

a) Find the equation of the tangent line when $x = 0$.

b) Graph both equations on your calculator in a standard viewing window. Is the tangent line a good approximation for the curve? Zoom in (at the origin) a couple of times. What do you notice?

c) Use the tangent line to approximate sin (0.2).

2. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

A 2.00

B 2.03

C 2.06

D 2.12

E 2.24

3. [With calculator] Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate the values of $f(x)$. Which of the following is the greatest value for which the error resulting from this tangent line approximation is less than 0.5?

A 2.4

B 2.5

C 2.6

D 2.7

E 2.8

4. Find the differential dy when $dx = -0.2$ and $x = 1$, if $y = x^2 e^x$. Explain what you've found.

5. Without a calculator, use differentials to approximate $\sqrt[4]{19}$.

6. [Calculator Required] The range R of a projectile is $R = \frac{{v_0}^2}{32}(\sin 2\theta)$, where ${v_0}^2$ is the initial velocity in feet per second and θ is the angle of elevation. Let $v_0 = 2200$ feet per second and let θ change from 10° to 11°.
[Remember … in calculus, we never use degrees!]

a) Find the actual change in the range.

b) Use differentials to *approximate* the change in the range. Are your answers "close" ?

7. Use linearization to approximate $f(0.1)$ if $f(x) = \frac{1}{\sqrt{4+x}}$. Find the error for your approximation.

8. If $y = \sin(x^2 - 3)$, find dy if $x = \sqrt{3}$ and $dx = \frac{1}{10}$.

9. The radius of a ball bearing is measured to be 0.7 inch. If the measurement is correct to within 0.01 inch, estimate the error in the volume of the ball bearing. [$V = \frac{4}{3}\pi r^3$]

10. How accurately should you measure the radius of a sphere in order to be reasonably sure the volume of the sphere is within 2% of its actual value? [Volume of a sphere: $V = \frac{4}{3}\pi r^3$]

11. A right circular cone has a radius that is one-third of the height. How accurately must the radius be measured so that error in calculating the volume is no more than 3% ? [Volume of a cone: $V = \frac{1}{3}\pi r^2 h$]

12. The circumference of the equator of a sphere is measured as 10 cm with a possible error of 0.4 cm. This measurement is then used to calculate the radius. The radius is then used to calculate the surface area and volume of the sphere. Estimate the percentage errors in the calculated values of the following:

a) the radius

b) the surface area [$S = 4\pi r^2$]

c) the volume

AP Calculus
4.6 Worksheet

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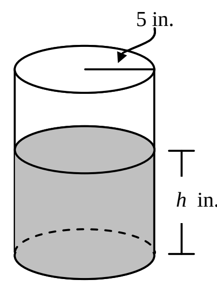
OK … I couldn't find a decent looney tunes picture for the next problem, so I thought I'd just throw in this cartoon (which by the way has nothing to do with related rates!) since I found it looking for any other good pictures. Besides, poor Wile E. Coyote has been working so much this year, it's about time he finally got a good meal. ☺

1. The radius r and area A of a circle are related by the equation: $A = \pi r^2$

Write an equation that relates $\frac{dA}{dt}$ and $\frac{dr}{dt}$.

2. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure to the right. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

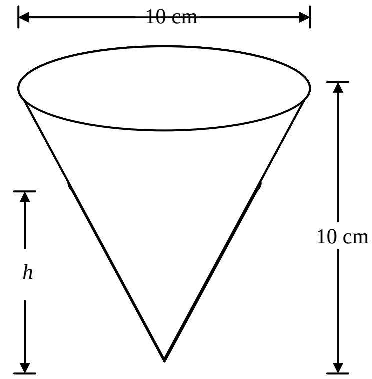


3. A 14 ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the end be moving away from the wall when the top is 6 ft above the ground?

4. An aircraft is climbing at a 45° angle to the horizontal. How fast is the aircraft gaining altitude if its horizontal speed is 400 mi/hr?

5. A container has the shape of an open right circular cone, as shown in the figure to the right. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth *h* is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(The volume of a cone of height *h* and radius *r* is given by $V = \frac{1}{3}\pi r^2 h$.)

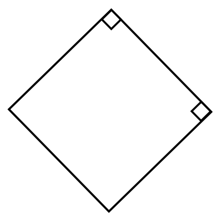


a) Find the volume *V* of water in the container when *h* = 5 cm. Indicate units of measure.

b) Find the rate of change of the volume of water in the container, with respect to time, when *h* = 5 cm. Indicate units of measure.

c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

6. A baseball diamond has the shape of a square with sides 90 feet long. Tweety is just flying around the bases, running from 2nd base (top of the diamond) to third base (left side of diamond) at a speed of 28 feet per second. When Tweety is 30 feet from third base, at what rate is Tweety's distance from home plate (bottom of diamond) changing?



7. A spherical container is deflated such that its radius decreases at a constant rate of 10 cm/min. At what rate must air be removed when the radius is 5 cm? [The volume of a sphere is $V = \frac{4}{3}\pi r^3$]

8. A pebble is dropped into a still pool and sends out a circular ripple whose radius increases at a constant rate of 4 ft/s. How fast is the area of the region enclosed by the ripple increasing at the end of 8 s?

9. Liquid is pouring through a cone shaped filter at a rate of 3 in^3/min. Assume that the height of the cone is 12 inches and the radius of the base of the cone is 3 inches. How rapidly is the depth of the liquid in the filter decreasing when the level is 6 inches deep?

10. Sand pours out of a chute into a conical pile whose height is always one half its diameter. If the height increases at a constant rate of 4 ft/min, at what rate is sand pouring from the chute when the pile is 15 ft high?

11. The radius r, height h, and volume V of a right circular cylinder are related by the equation $V = \pi r^2 h$.

a) How is $\frac{dV}{dt}$ related to $\frac{dh}{dt}$ if r is constant?

b) How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ if h is constant?

c) How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ and $\frac{dh}{dt}$ if neither r nor h is constant?

12. Complete #42 on page 254 of your textbook.

AP Calculus
5.1 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The function f is continuous on the closed interval [2, 8] and has values that are given in the table below.

x	2	5	7	8
$f(x)$	10	30	40	20

Using the subintervals [2, 5], [5, 7], and [7, 8], what are the following approximations of the area under the curve?

A) LRAM

B) RRAM

C) MRAM

D) Trapezoid Rule

2. Complete the following problems from the textbook:
pages 270 – 273: #2, 4, 11 (use 4 rectangles and any RAM), 17, 23, 30, 34, 35, 36

AP Calculus
5.2 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

An Activity: Instead of using LRAM and RRAM, let's introduce a "lower" and "upper" estimate to use for this example. A lower estimate, uses the lowest *y*-value in an interval regardless of whether this point is on the left side or the right side. Similarly, an upper estimate uses the highest *y*-value in an interval.

A car is traveling so that its speed is never decreasing during a 10 – second interval. The speed at various moments in time is listed in the table below.

Time (sec)	0	2	4	6	8	10
Speed (ft/sec)	30	36	40	48	54	60

a) Explain why the best lower estimate for the distance traveled in the first 2 seconds is 60 feet.

b) Explain why the best upper estimate for the distance traveled in the first 2 seconds is 72 feet.

c) Find the best lower estimate for the distance traveled in the first 10 seconds.
♬: An answer of 300 feet (which ignores some of the data) is not correct.

d) Find the best upper estimate for the distance traveled in the first 10 seconds.
♬: An answer of 600 feet (which ignores some of the data) is not correct.

… These sums of products that you have found in *c* and *d* are called ___________________.

e) If you choose the lower estimate for your approximation of how far the car travels, what is the maximum amount your approximation could differ from the exact distance? … In other words, how far apart are your estimates?

f) Choose speeds to correspond with t = 1, 3, 5, 7, and 9 seconds. Keep the nondecreasing nature of the above table and do not select the average of the consecutive speeds. Find new best upper and lower estimates for the distance traveled for these 10 seconds. … how far apart are your estimates?

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Speed (ft/sec)	30		36		40		48		54		60

New Upper Estimate: _______ New Lower Estimate: _________

g) Compare how far apart your upper and lower estimates are with at least two other groups/individuals who didn't use the same numbers as you did. What do you notice?

h) Make a prediction … How far apart do you think the estimates would be if we extended the last table to include speeds that correspond with t = 0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7,5, 8.5, and 9.5 seconds?

i) If we continue to introduce more entries into our charts, what happens to the upper and lower estimates?

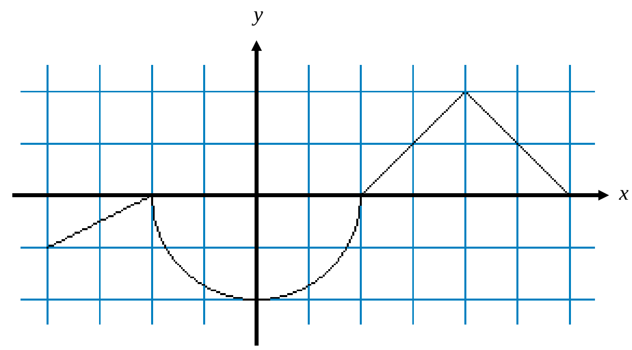
h) Write an expression giving the ACTUAL distance this car traveled in 10 seconds, if it's velocity was $v(t)$.

1. Complete the following questions from your textbook: page 282 # 1 – 9 (odd), 10 – 22 (even), 23, 25, 30, 32, 37, 39, 41 - 43

AP Calculus
5.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The graph of f shown below consists of line segments and a semicircle. Evaluate each definite integral.



a) $\int_0^2 f(x)\,dx$

b) $\int_2^6 f(x)\,dx$

c) $\int_{-4}^2 f(x)\,dx$

d) $\int_{-4}^6 f(x)\,dx$

e) $\int_{-4}^6 |f(x)|\,dx$

f) $\int_{-4}^6 [f(x)+2]\,dx$

2. Part *e* above, gives a way to find the total Area between the x – axis and the function between $x = -4$ and $x = 6$. Without using absolute value signs, write two different expressions that can be used to find the total area between the x – axis and the function between $x = -4$ and $x = 6$.

3. [Calculator Required … for now] What is the average value of $y = x^2\sqrt{x^3+1}$ on [0, 2] ?

4. [Calculator Required] … Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

a) Is traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

b) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

c) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure

5. [Calculator Required] At different altitudes in Earth's atmosphere, sound travels at different speeds. The speed of sound $s(x)$ (in meters per second) can be modeled by

$$s(x)=\begin{cases} -4x+341 & \text{if } 0\le x<11.5 \\ 295 & \text{if } 11.5\le x<22 \\ \frac{3}{4}x+278.5 & \text{if } 22\le x<32 \\ \frac{3}{2}x+254.5 & \text{if } 32\le x<50 \\ -\frac{3}{2}x+404.5 & \text{if } 50\le x\le 80 \end{cases}$$

where x is measured in kilometers. What is the average speed of sound over the interval [0. 80]?

6. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table below gives the measurements of the diameter of the blook vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel, and $B(x)$ is a twice differentiable function that represents the diameter at that point.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

b) Approximate the value of your answer from part *a* using the data from the table and a midpoint Riemann Sum with three subintervals of equal length. Show the computations that lead to your answer.

c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

7. Complete the following questions from the textbook:

pages 290 – 292 #1, 4, 5, 11 – 16, 40, 41, 47, 48, 49 … AND … p293 #1, 2

(for #11 – 14 … NINT means to use your calculator to integrate)

AP Calculus
5.4 Worksheet

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FTOC: The Evaluation Part

1. Complete the following questions from your textbook: Page 303 # 27, 29, 30, 35, 37 – 40, 43, 45

FTOC: The Derivative of an Integral Part (Simple)

2. Complete the following questions from your textbook: Page 302 #2, 3, 4, 6

FTOC: The Derivative of an Integral Part (Extended)

3. Find $\frac{d}{dx}\left[\int_{1}^{\sin x} \sqrt{1+t^3}\, dt\right]$.

4. Find $\frac{d}{dx}\left[\int_{\sin x}^{x^3} f(t)\, dt\right]$

5. Find $\frac{d}{dx}\left[\int_{\sin x}^{x^3} e^{t^2}\, dt\right]$

6. Complete the following questions from your textbook: Page 302 #11, 17, 28

Putting it all together …

7. Complete the following questions from your textbook: Page 303 #57, 59

8. What are all the values of *k* for which $\int_{2}^{k} x^2\, dx = 0$?

A –2

B 0

C 2

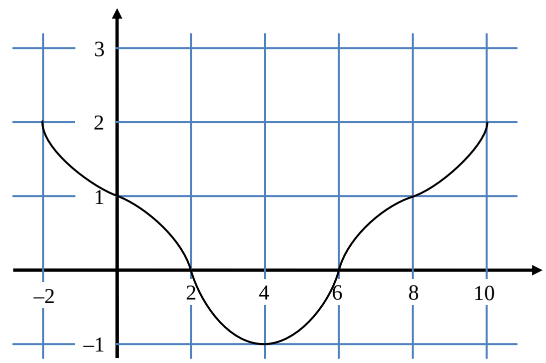
D –2 and 2

E –2, 0, and 2

9. The graph of a differentiable function f on the interval [–2, 10] is shown in the figure below. The graph of f has a horizontal tangent line at $x = 4$. Let $h(x) = 9 + \int_4^x f(t)\,dt$ for $-2 < x < 10$.

a) Find $h(4)$, $h'(4)$, and $h''(4)$

Graph of *f*



b) On what intervals is h increasing? Justify your answer.

c) On what intervals is h concave downward? Justify your answer.

d) Find the Trapezoidal Sum to approximate $\int_{-2}^{10} f(x)\,dx$ using 6 subintervals of length = 2.

10. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function *r* of time *t*, where *t* is measured in minutes. For $0 < t < 12$, the graph of *r* is concave down. The table below gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (The Volume of a sphere is given by $V = \frac{4}{3}\pi r^3$)

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

c) Use a right Riemann Sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

d) Is your approximation is part *c* greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.